Insight Gained from Simplified Dynamic Analysis

... or Everything Old is New Again

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Greg Mertz
Consultant

Why Simplified Models?

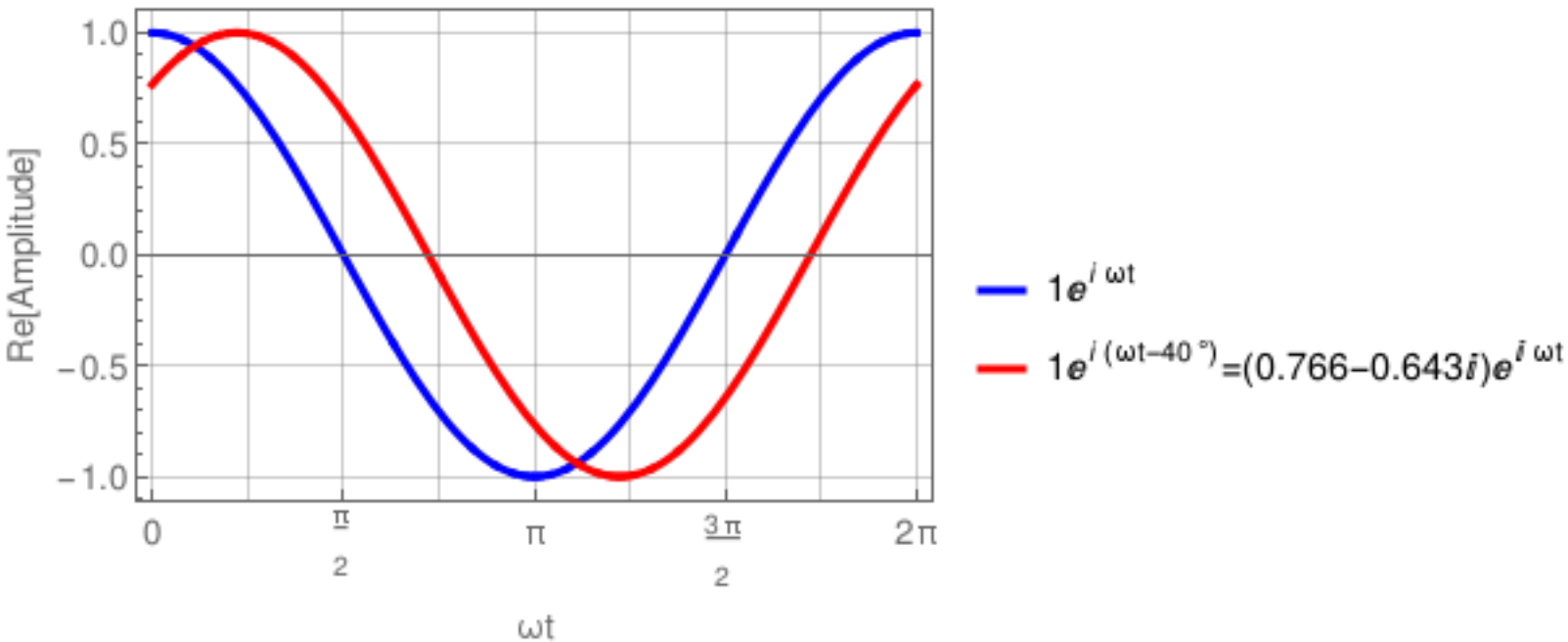
- Provide valuable insight into complex behavior
  - Forces you to identify load path
    - Simple models focus on the load path
  - Bypass analysis clutter
    - Focus on primary load path
  - Understand sensitivity to different inputs
- Economical
  - Cheap compared to building a FEM model, running the model and interpreting results
- Independent check on results
  - Avoid GIGO

Methodology

- Static Analysis
  - $[K]\{x\} = \{P\}$
  - Solve for unknown displacements, x, or loads, P
- Steady State Dynamics
  - $(-\omega^2[M] + i\omega[C] + [K])\{x\}e^{i\omega t} = \{P\}e^{i\omega t}$
    - $[S]\{x\}=\{P\}$
  - Solve for unknown displacements or loads at each frequency, ω
    - Interpolate intermediate frequencies
  - Stiffness may include complex damping
  - Complex results

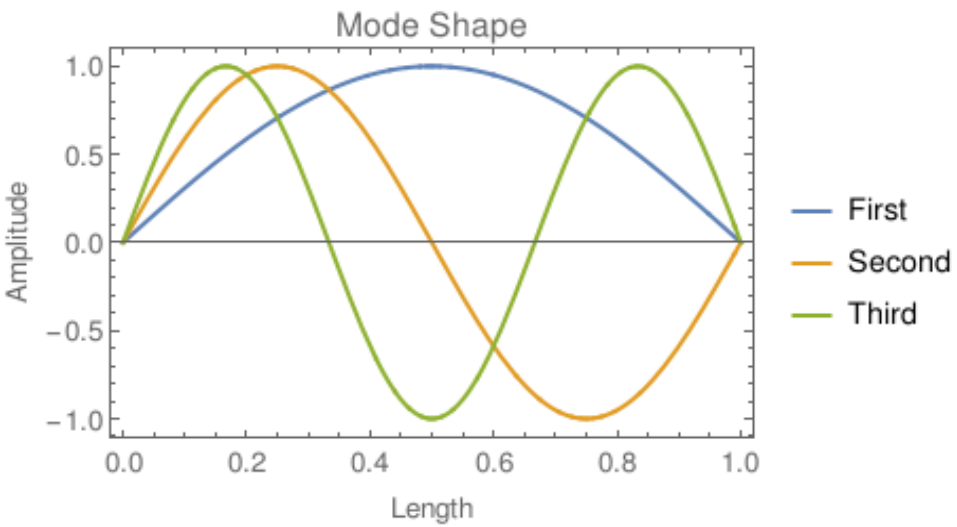
Methodology (cont)

- Steady State load and displacements
  - Input and results are complex numbers
    - $(a+ib)\, e^{i\omega t} = Ae^{i(\omega t+\Phi)} = A(\mathrm{Cos}(\omega t+\Phi)+i\cdot\mathrm{Sin}(\omega t+\Phi))$

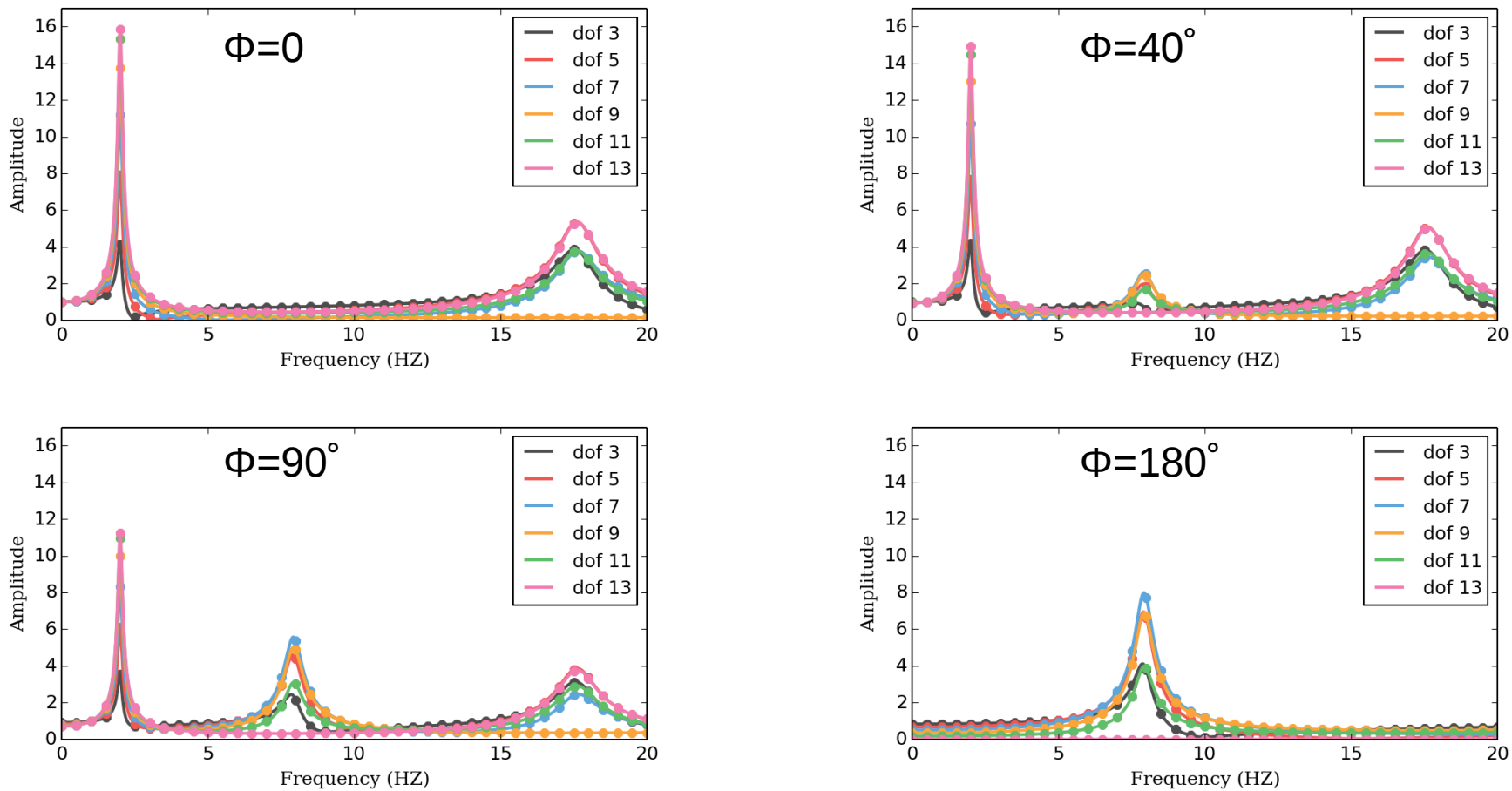


Simple Span Beam

- Can vertical seismic excite the second mode of a beam?
- Beam properties
  - fn= 2, 8, 18, ... Hz
  - 4% Damping



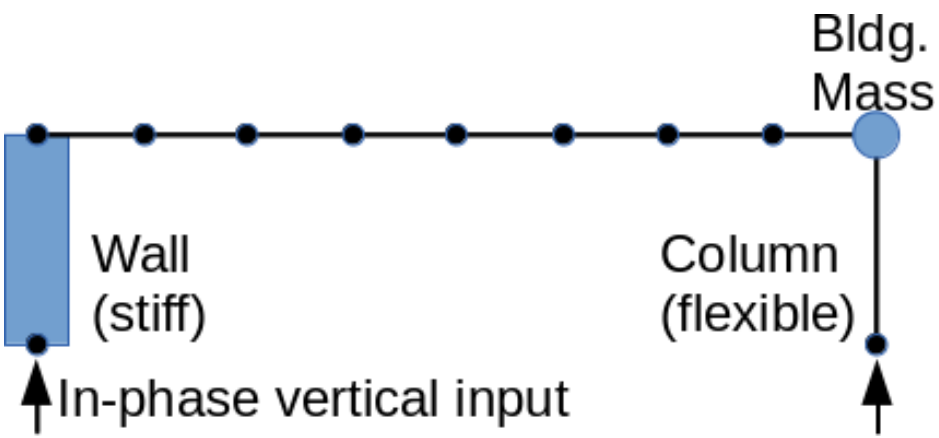
Beam Transfer Functions

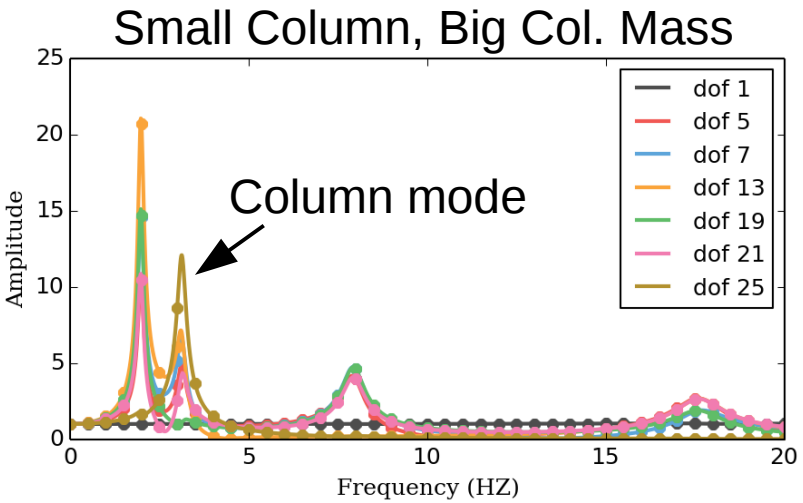


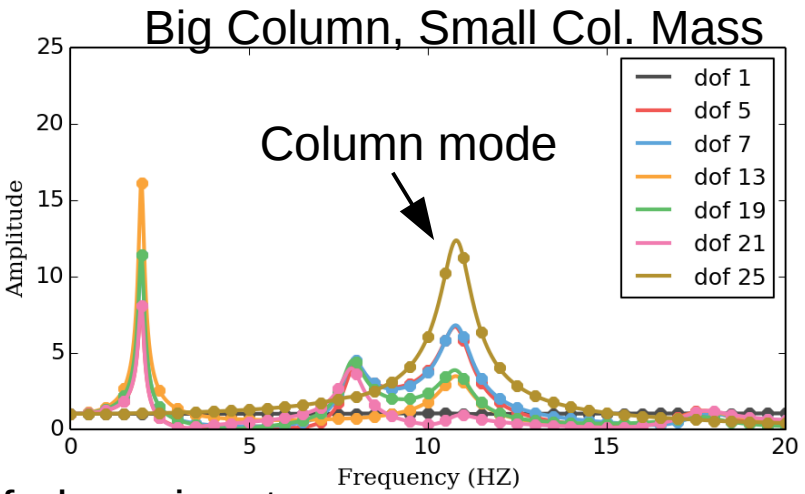
Out of phase input (1) reduces odd mode response; (2) increases even mode response

Beam in a Building

- Include wall and column support flexibility
  - In-phase input
  - Column has additional building mass



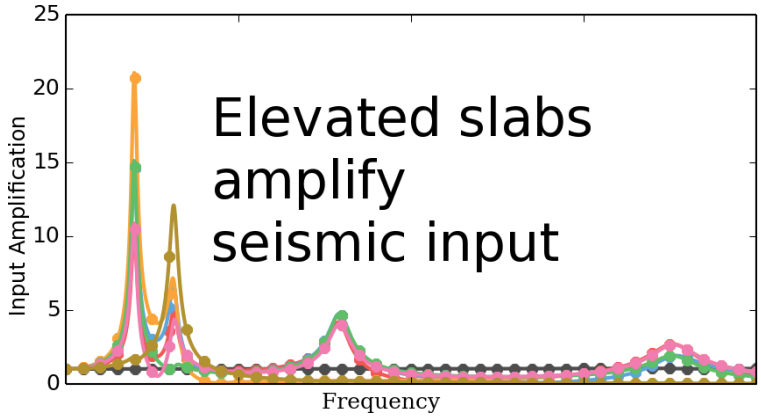


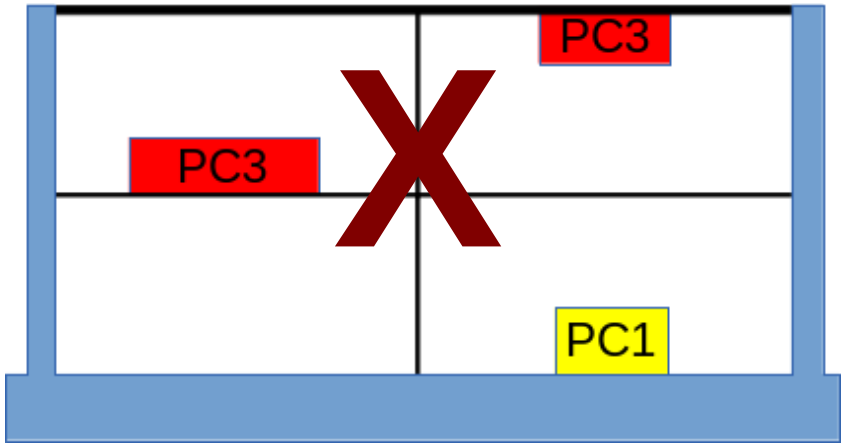


Different support flexibility can cause out-of-phase input.
Support flexibility may amplify response.

Insight / Opinion

- Why would you ever put <u>important</u> equipment on an elevated slab in a <u>new design</u>?

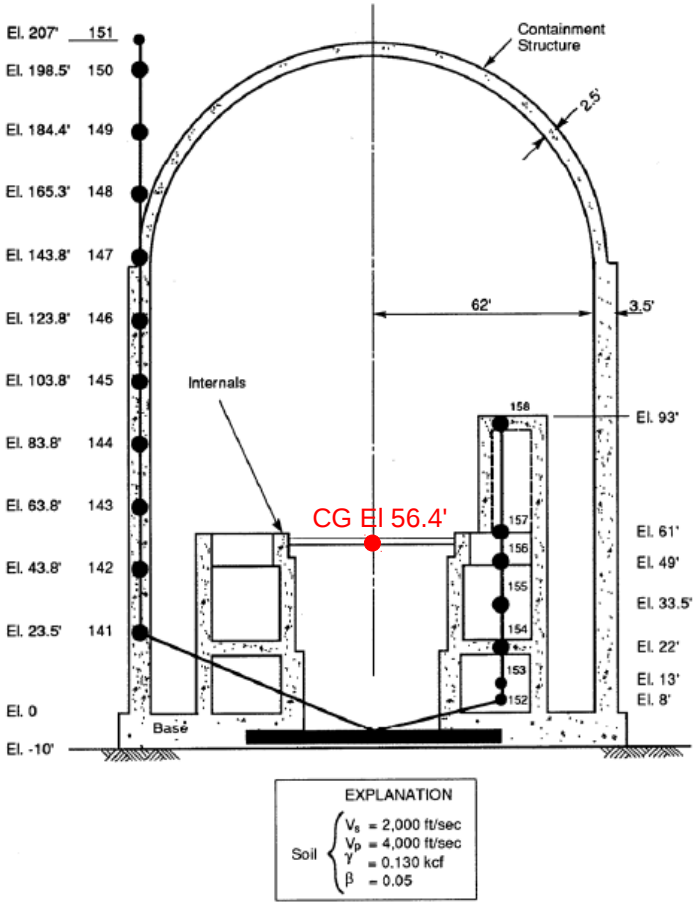




- Put important equipment on the basemat!

Rigid Body SSI → Simplified SSI

- Goal: Understand the impact of SSI on building response
  - Lateral direction
- Stages
  - Lateral RB SSI
  - Lateral + Rocking RB SSI
  - FB Building
  - Lateral + Rocking + Bldg



Ref: SASSI UM

Refresher

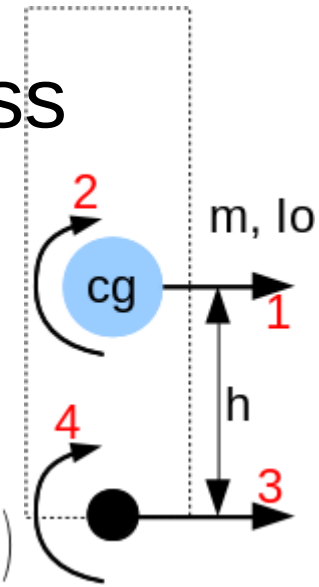
- ASCE 4 Soil Springs

TABLE 3.3-1. Lumped Representation of Structure-Foundation Interaction at Surface for Circular Base

Motion	Equivalent Spring Constant	Equivalent Damping Coefficient
Horizontal	$k_x = \frac{32(1-\nu)GR}{7-8\nu}$	$c_x = 0.576 k_x R\sqrt{\rho/G}$
Rocking	$k_\psi = \frac{8GR^3}{3(1-\nu)}$	$c_\psi = \frac{0.30}{1+B_\psi} k_\psi R\sqrt{\rho/G}$
Vertical	$k_z = \frac{4GR}{1-\nu}$	$c_z = 0.85 k_z R\sqrt{\rho/G}$
Torsion	$k_t = 16GR^3/3$	$c_t = \frac{\sqrt{k_t I_t}}{1+2I_t/\rho R^5}$

Notes: ν = Poisson's ratio of foundation medium; G = shear modulus of foundation medium; R = radius of circular basemat; ρ = mass density of foundation medium; $B_\psi = 3(1-\nu)I_o/8\rho R^5$; I_o = total mass moment of inertia of structure and basemat about the rocking axis at the base; and I_t = polar mass moment of inertia of structure and basemat.

- Rigid Body Mass



From kinematics

$$x1 = x3 + h\,x4$$

$$x2 = x4$$

or in matrix form

$$\begin{pmatrix} x1 \\ x2 \end{pmatrix} = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x3 \\ x4 \end{pmatrix} = T \begin{pmatrix} x3 \\ x4 \end{pmatrix}$$

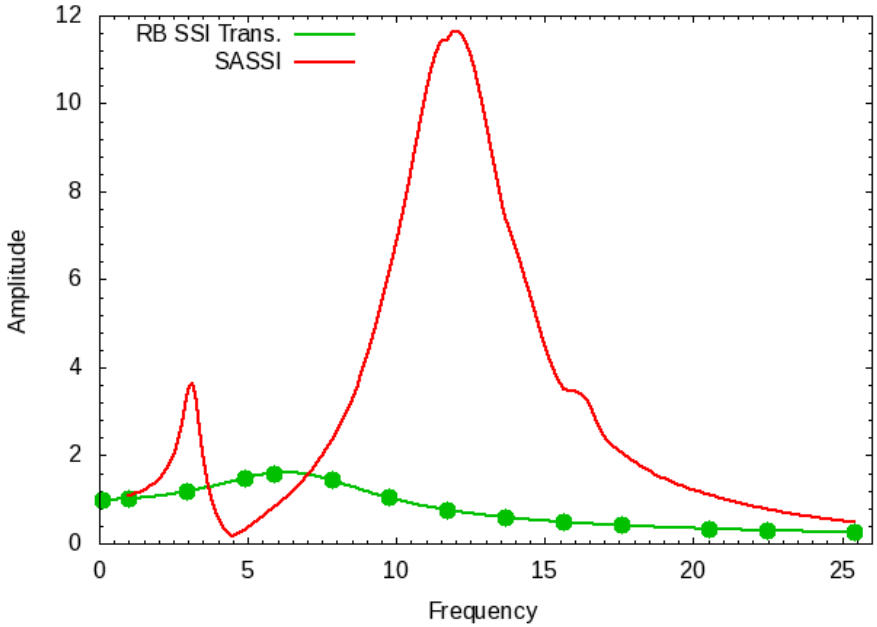
The mass matrix at the CG, in terms of DOF 1 and 2, is

$$M_{CG} = \begin{pmatrix} m & 0 \\ 0 & Icg \end{pmatrix}$$

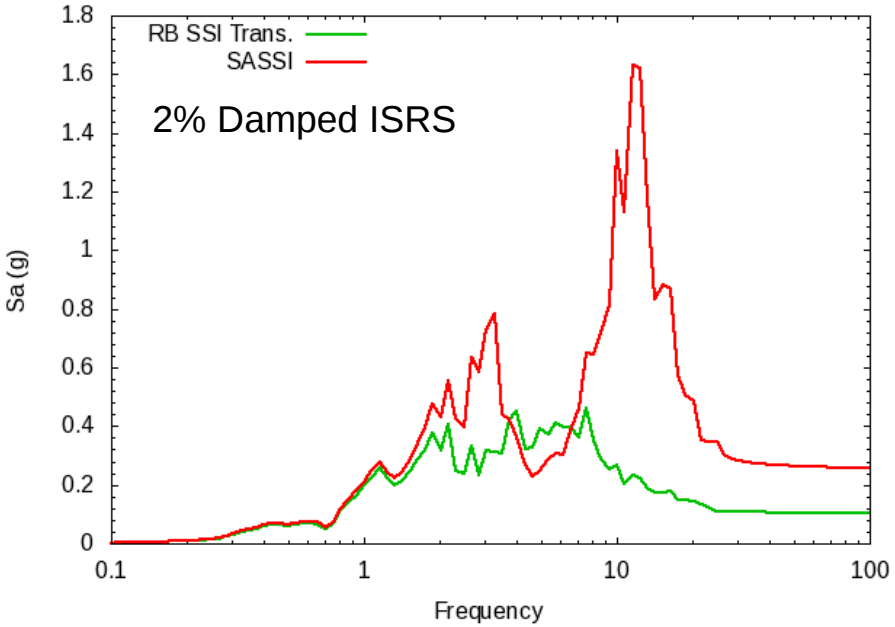
Transfer the mass to DOF 3 and 4 by

$$M_B = T^T M_{CG}\, T = \begin{pmatrix} m & h\,m \\ h\,m & Icg + h^2\,m \end{pmatrix}$$

Rigid Body SSI ASCE 4 Spring Translation Only

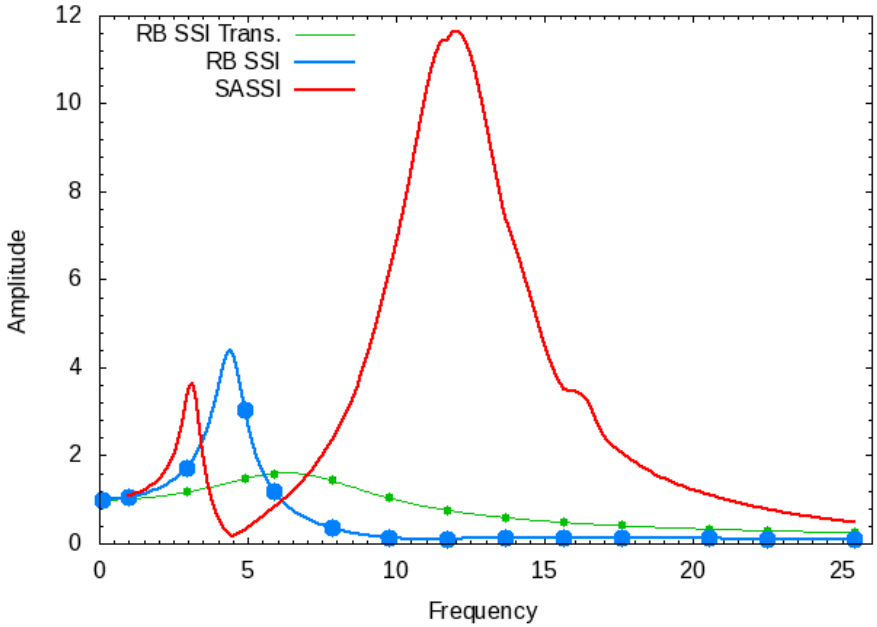


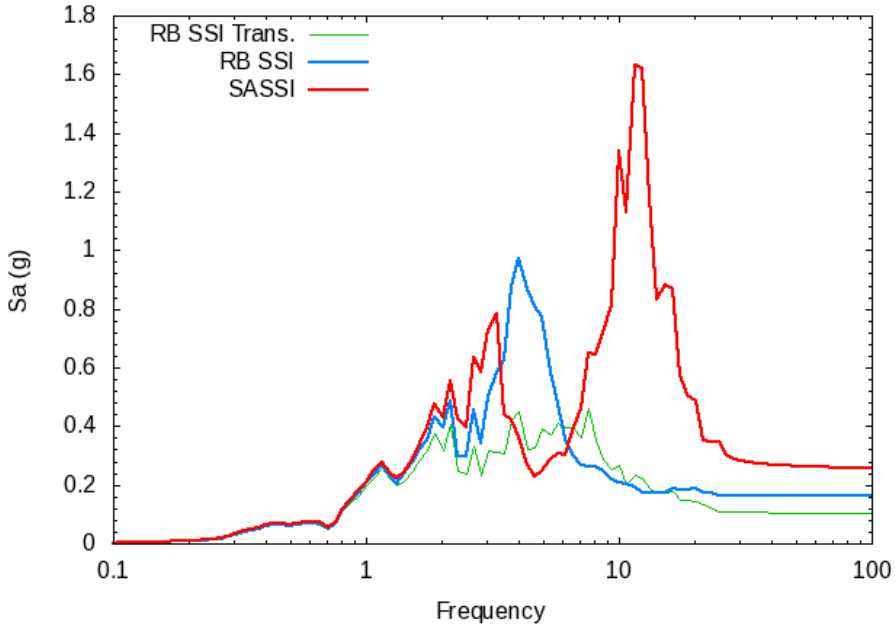
Response at top of internals



- Lateral SSI alone is not a strong influence in this problem

Rigid Body SSI
ASCE 4 Spring
Translation+Rocking

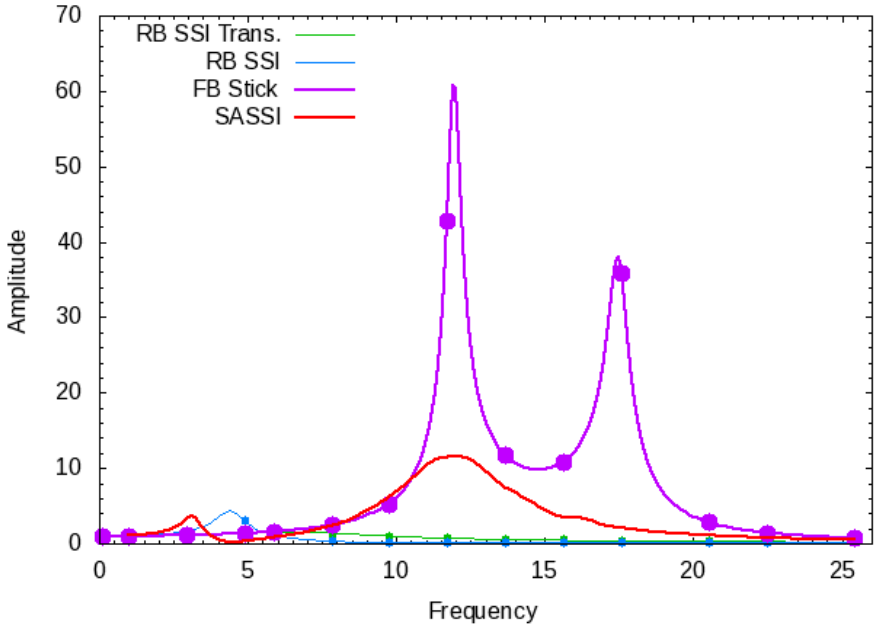


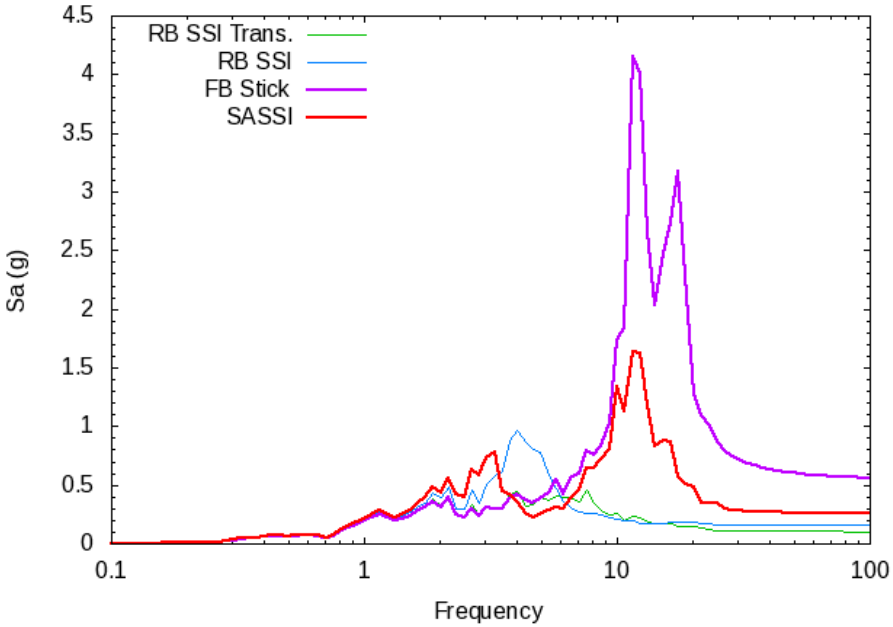


Response at top of internals

- Response near 3 Hz appears to be a rocking mode
- Significant reduction in input above 8 Hz

Fixed Base Stick



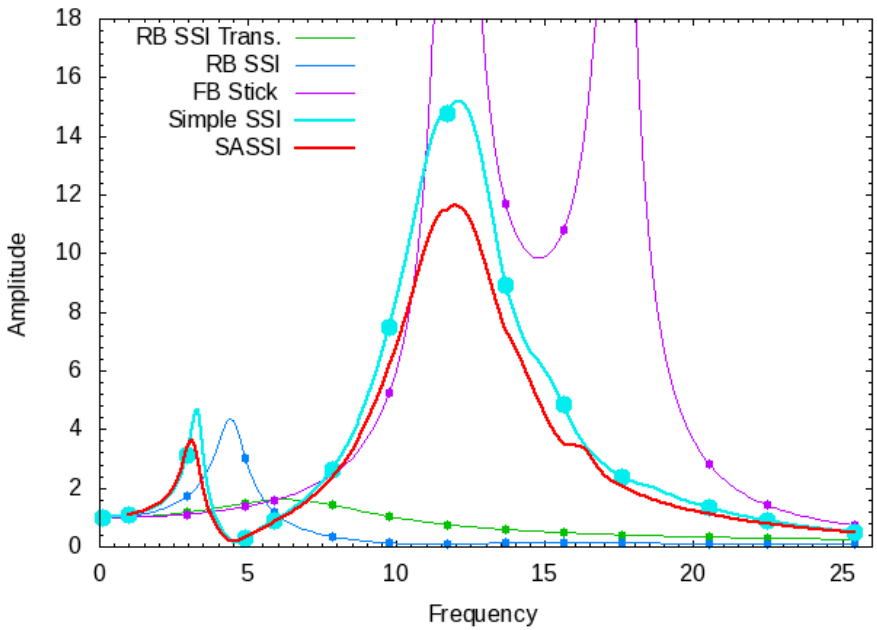


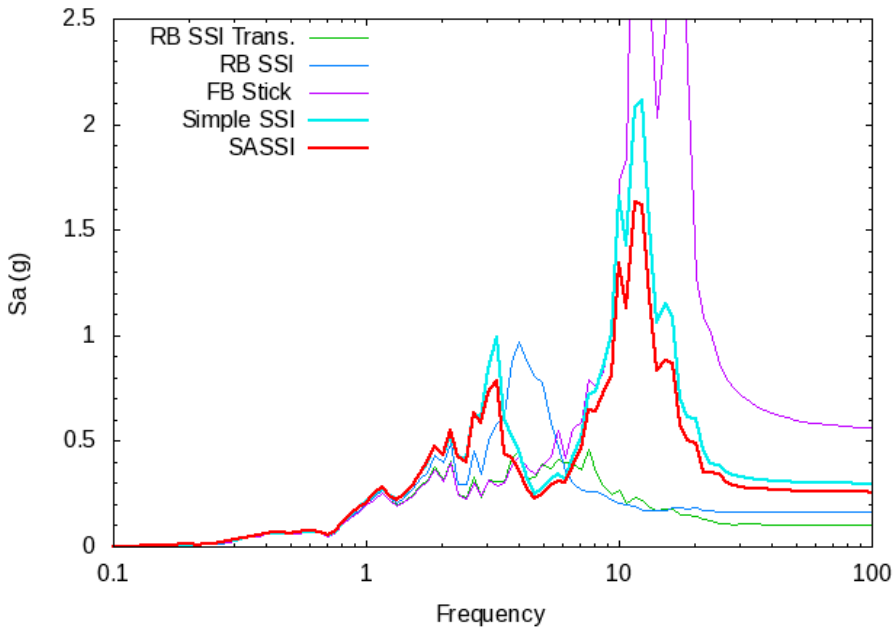
Response at top of internals

2% Damping in structure

- Response near 12 Hz appears to be structural mode
- SSI reduces structural mode

ASCE 4 Soil Springs with Stick

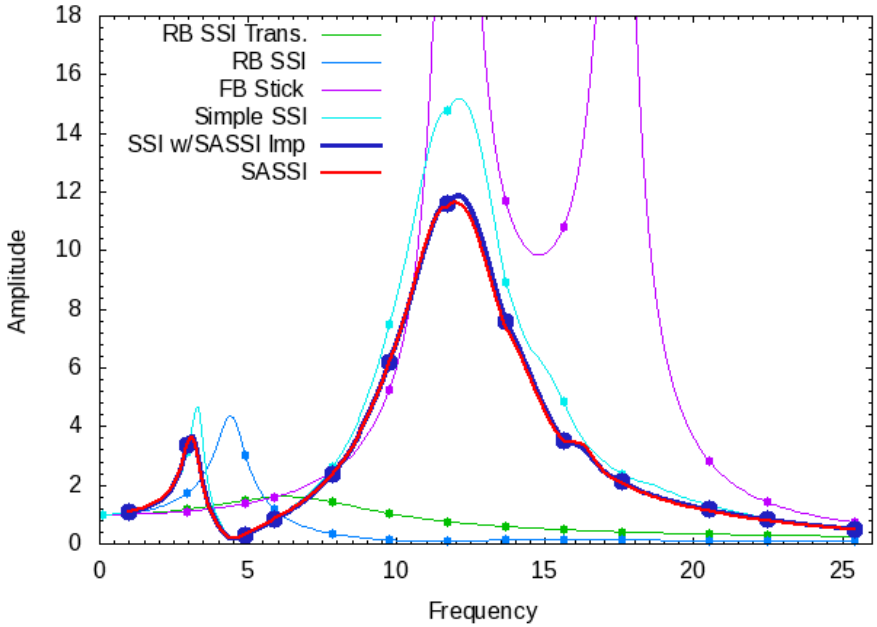


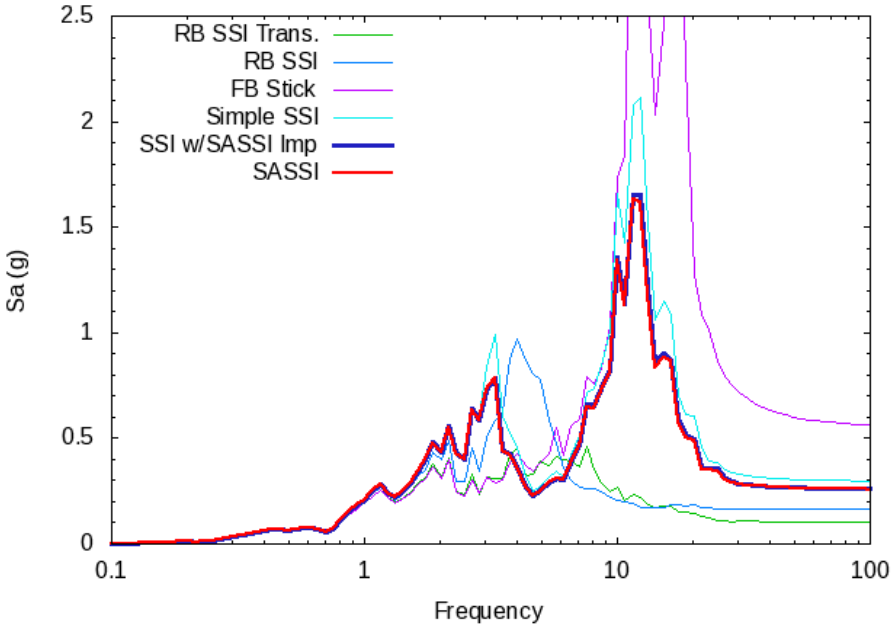


Response at top of internals

- ASCE 4 soil springs + stick yield:
  - 130% of the peak Sa
  - 115% of ZPA
- Simple models are often able to represent a significant portion of the 'correct' answer.

SASSI Impedance Soil Springs with Stick





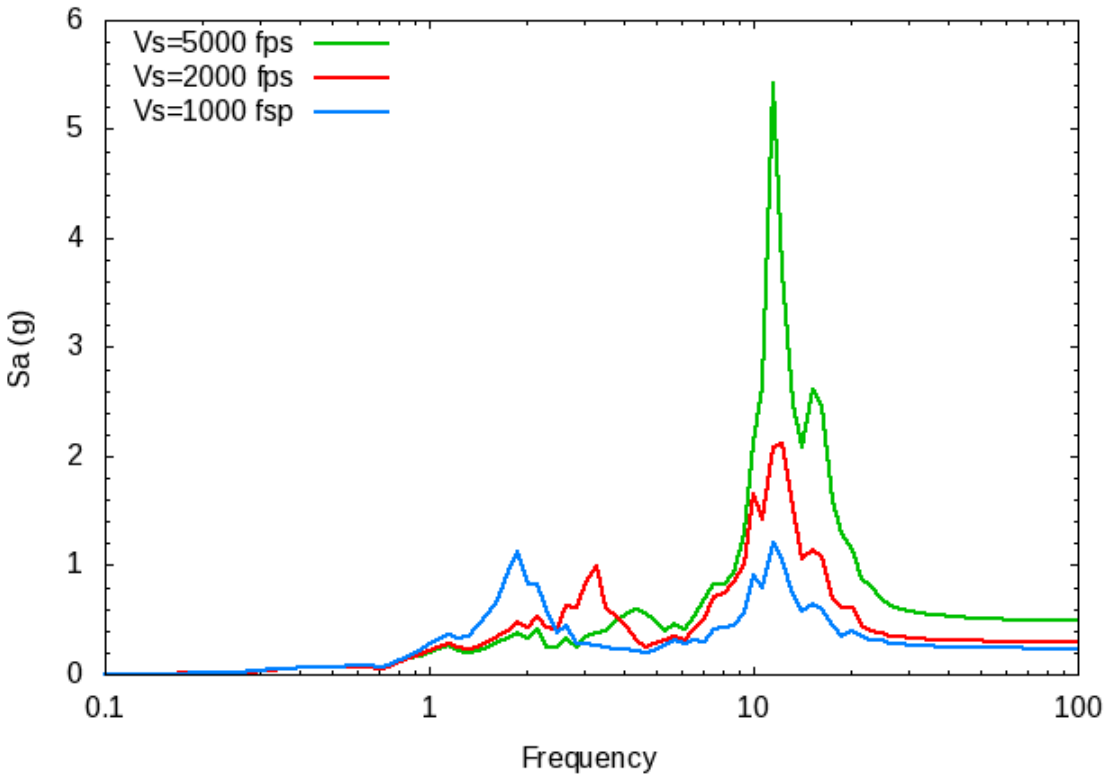
Response at top of internals

Included coupled translation-rocking impedance term

- Inclusion of complex soil model
  - reduces peak Sa ≈25%
  - reduces ZPA ≈15%

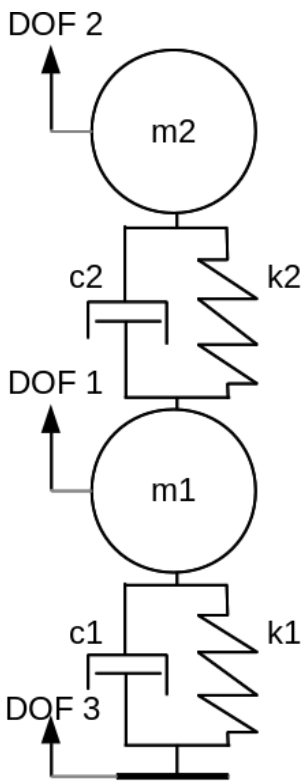
Sensitivity Study

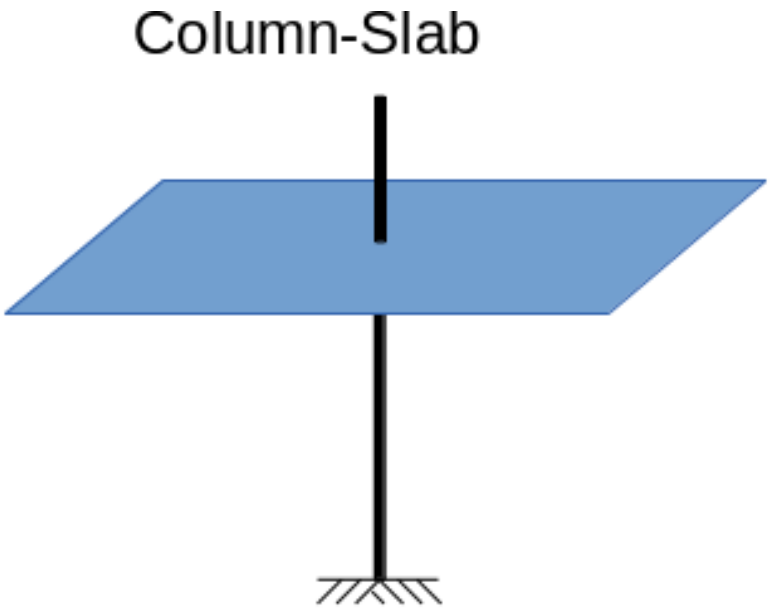
- SSI Translation and Rocking reduced response
  - Site: Vs=2000 fps
- Evaluate other sites
  - SRS Vs≈1000 fps
  - ORNL Vs≈5000 fps
- ASCE 4 Soil Springs

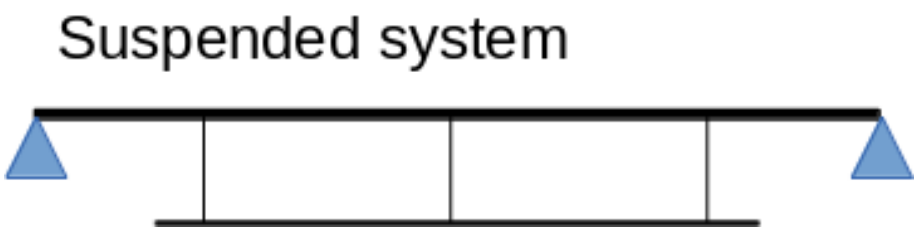


Secondary System Response

- Coupled primary and secondary system

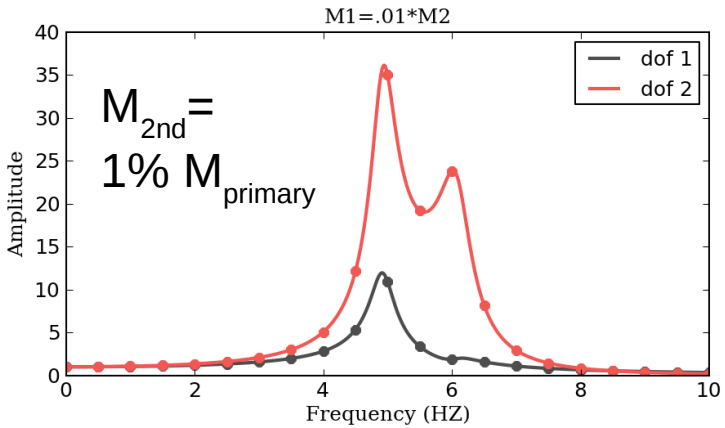


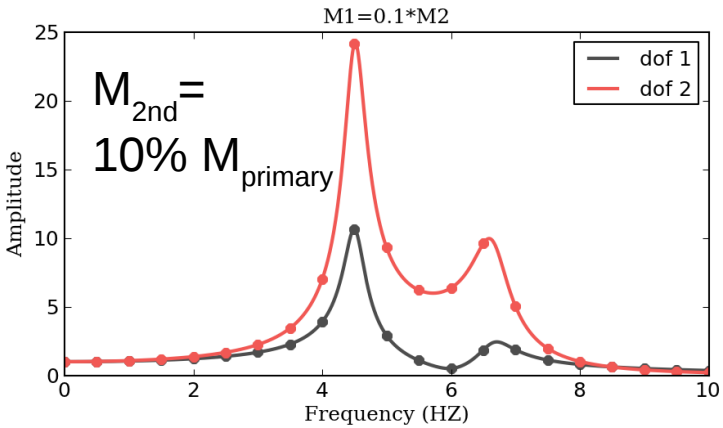


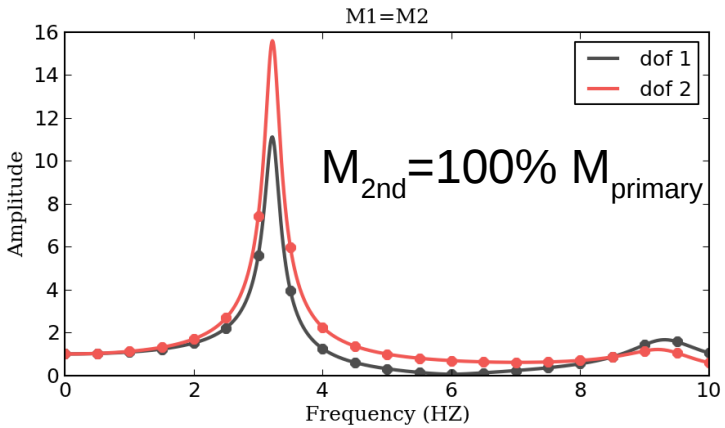


2 DOF Interaction

- 6 Hz secondary system
- 5 Hz primary system
- 4% damping







Methodology - Tool Box

- Calculations performed using Python program ERICA
- Pre-release freeware
  - Estimated release late December
  - use, learn, modify, share
    - or ignore...
  - Not NQA
  - Not intended to replace commercial codes
  - Intended to develop insight

```
# Case A, m2=0.01 m1
# ------------------
# define variables for a mass-spring-dashpot dynamic system

# define primary system between dof 3 and 1
 m1 = 1.0                      # assumed mass
 k1 = m1*(2*pi*5)^2            # stiffness for 5 Hz and m1
 rho= 0.04                     # damping ratio
 damp=1.0-2.*rho^2+ \
     2j*rho*np.sqrt(1-rho^2) # factor for complex damping

 spring, dof=[3,1],value=k1*damp
 mass,   dof=1,    value=m1

# define secondary system between dof 1 and 2
 m2 = 0.01*m1                  # case A mass
 k2 = m2*(2*pi*6)^2            # stiffness for 6 Hz and m2
 c2 = rho*2.0*np.sqrt(m2*k2) # secondary dashpot

 spring, dof=[1,2],value=k2
 dashpot,dof=[1,2],value=c2
 mass,   dof=2,    value=m2

# impose a unit base displacement on dof 3
 displacement,dof=3,value=1.0+0j

# define freq. for explicit soln, say 0-10 Hz in 0.5 Hz inc.
# also define an interpolated frequency step of 0.01 hz
 analysis,freq=[float(x*0.5) for x in range(21)],df=0.01

# get interpolated transfer function for dof  1 and 2
 plot,var=tf,file=p1-a,dof=[1,2],popt title=M1=0.01*M2
```

Summary

- Simplified Dynamic Analysis
  - Can represent a significant portion of full solution
  - Provide valuable insight
  - Allows cost effective sensitivity studies
  - Provide confidence in full solution

- Put important equipment on the basemat