Efficient AC Optimal Power Flow & Global Optimizer Solutions

CERTS Review meeting, 8/06/2014
Cornell University, Ithaca, New York

Zhe Hu, Göktürk Poyrazoğlu, and HyungSeon Oh
University at Buffalo

CERTS

Overview

- Optimal power flow
- Polar vs. Cartesian coordinate system
- Global optimizer
- Research idea
- Our proposed method
- Simulation result
- Conclusions

Optimal Power Flow I

- Finds an optimal solution while satisfying
  - Kirchhoff's laws
  - Operation constraints
  - Economic, policy, and/or environmental constraints
- Plays a key role in operation and planning of
  - Smart grid technologies
  - Renewable energy integration
- Nonlinear and nonconvex → difficult to solve
  - No guarantee to find a solution
  - Heuristic search aiming for a local solution

Optimal Power Flow II

OPF in the polar coordinate system

- Control variables
  - Voltage magnitude (E) and angle (θ)
  - Real (p) and reactive (q) power generation

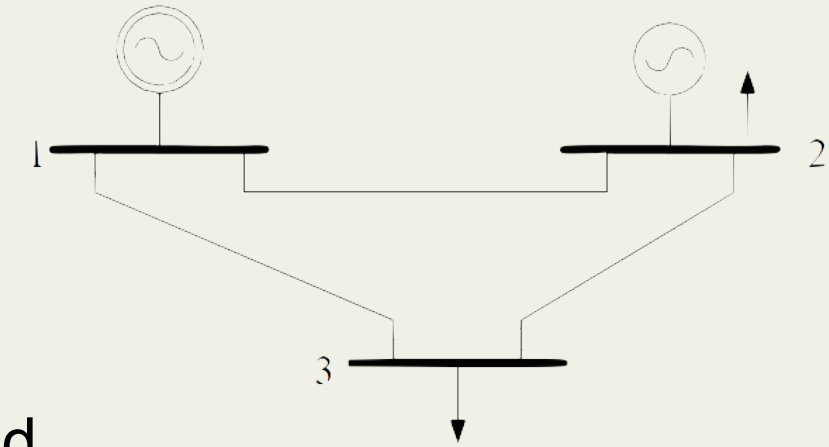
$$\min_{E,\theta,p,q} \sum_{i=1}^{N_g} \left[Cost_p^i\left(p_i\right) + Cost_q^i\left(q_i\right) \right]$$

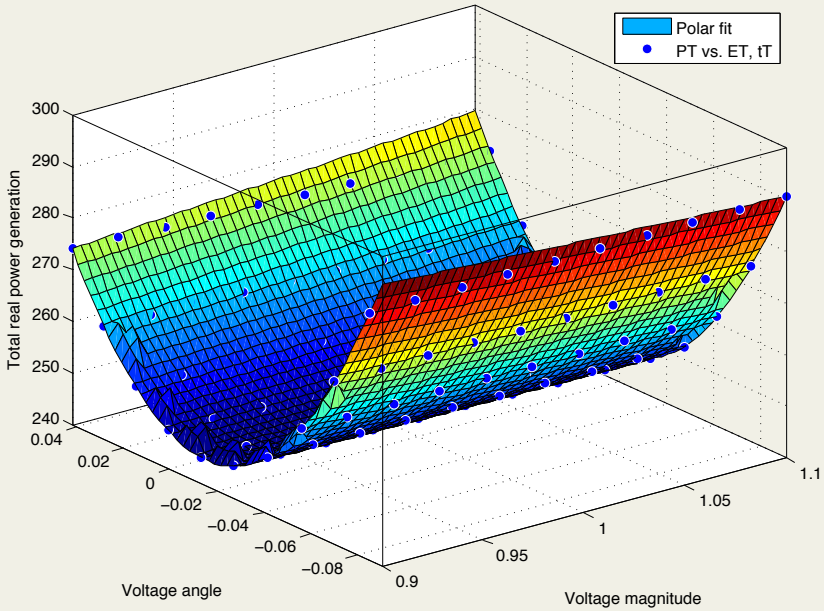
$$s.t. \begin{cases} g_p\left(E,\theta,p\right) = 0; \quad g_q\left(E,\theta,q\right) = 0 \\ flow\left(E,\theta\right) \le 0; \ \underline{E} \le E \le \overline{E} \\ \underline{p} \le p \le \overline{p}; \ \underline{q} \le q \le \overline{q} \end{cases}$$

- MATPOWER finds a local solution
  - Often observes $E \sim 1.0$ and $-\pi/3 \le \theta \le \pi/3$

Polar vs. Cartesian Coordinate System I

- Power balance equations are sinusoidal function with θ
- 3-bus system:
  - Bus 1: reference bus
  - Bus 2: PV bus
- Power flow solutions at various $|v|$ and power generation at Bus 2
- Power generation is a function of $|v|$ and θ at Bus 2
  - $|v|$ and θ are independent
  - Efficient search in the space of $|v|$ and θ is possible





Optimal Power Flow III

OPF in the Cartesian coordinate system

- Control variables
  - Real (x) and imaginary (y) components of voltages
  - Real (p) and reactive (q) power generation

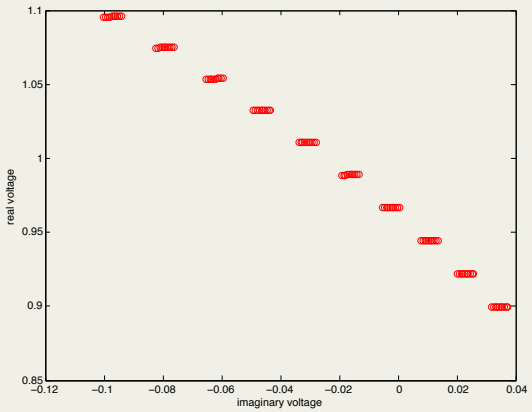
$$\min_{x,y,p,q} \sum_{i=1}^{N_g} \left[Cost_p^i\left(p_i\right) + Cost_q^i\left(q_i\right) \right]$$

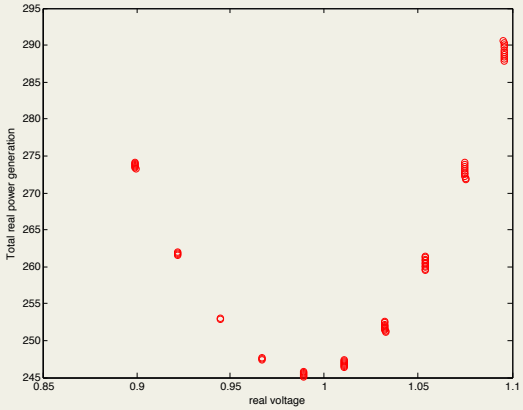
$$s.t. \begin{cases} g_p\left(x, y, p\right) = 0; \quad g_q\left(x, y, q\right) = 0 \\ flow\left(x, y\right) \le 0; \ \underline{E}^2 \le E\left(x, y\right) \le \overline{E}^2 \\ \underline{p} \le p \le \overline{p}; \ \underline{q} \le q \le \overline{q} \end{cases}$$

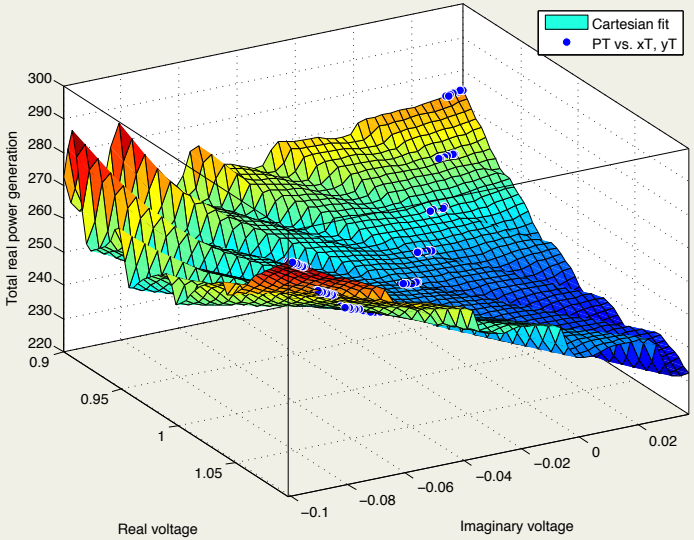
  - E and g are quadratic functions in x and y
  - *flow* is a quartet function in x and y

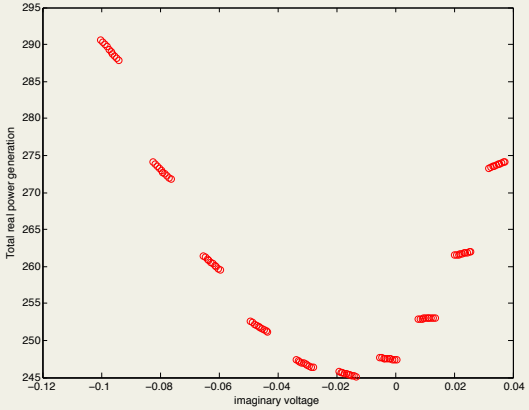
Polar vs. Cartesian Coordinate System II

- Power balance equations are quadratic in *x* and *y*
- *x* and *y* are dependent
- Saddle points?
- Different strategy for search









Optimal Power Flow IV

- Semi-definite programming (SDP) relaxes non-convexity

$$\min_{W,p,q} \sum_{i=1}^{N_g} \left[Cost_p^i(p_i) + Cost_q^i(q_i) \right]$$

$$s.t. \begin{cases} p_g^i - p_d^i = Tr\left(\Phi_p^i W\right); q_g^i - q_d^i = Tr\left(\Phi_q^i W\right); \\ \underline{E_i}^2 \le Tr\left(\Pi_i W\right) \le \overline{E_i}^2; \ \underline{p} \le p \le \overline{p}; \ \underline{q} \le q \le \overline{q}; \ W \succeq 0 \end{cases} \quad \& \begin{bmatrix} -\overline{flow_l}^2 & Tr\left(Y_l W\right) & Tr\left(\tilde{Y}_l W\right) \\ Tr\left(Y_l W\right) & -1 & 0 \\ Tr\left(\tilde{Y}_l W\right) & 0 & -1 \end{bmatrix} \preceq 0$$

  - $W = [x^T \ y^T]^T [x^T \ y^T]$
  - Rank(W) = 1 is relaxed with $W \ge 0$
  - If solution W is rank-1 → global solution
  - If not
    - In other application areas $W \approx \lambda^2 q q^T$
    - In power systems, no physically meaningful solution
- SDP yields a lower bound of all local minimizers

Global Optimizer I

Algorithms seeking for the global optimizer

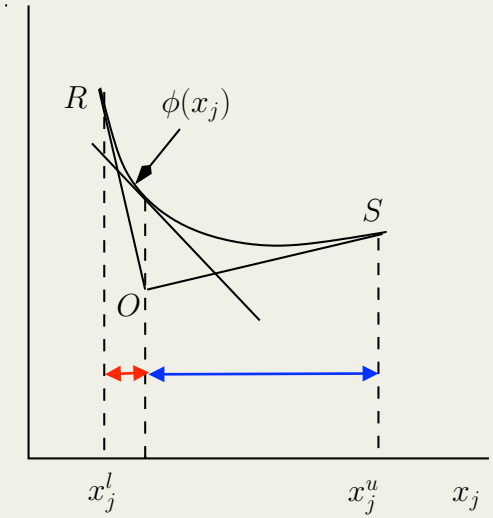
- Stochastic methods
  - Simulated annealing
  - Direct Monte-Carlo sampling
  - Stochastic tunneling
- Deterministic methods
  - Cutting plane method, and
  - Branch-and-bound method
- Our goal is to guarantee to find the global optimizer if exists

Global Optimizer II

Branch-and-reduce (BR)

- BARON is a widely used commercial software package to find the global solution
- OPF in the polar coordinate system cannot be solved with BARON because it does not take sinusoidal functions
- Convexification through a linear approximation
- Rule for selecting supporting lines in sandwich algorithm
  - Maximum error rule
  - Two sub-regions

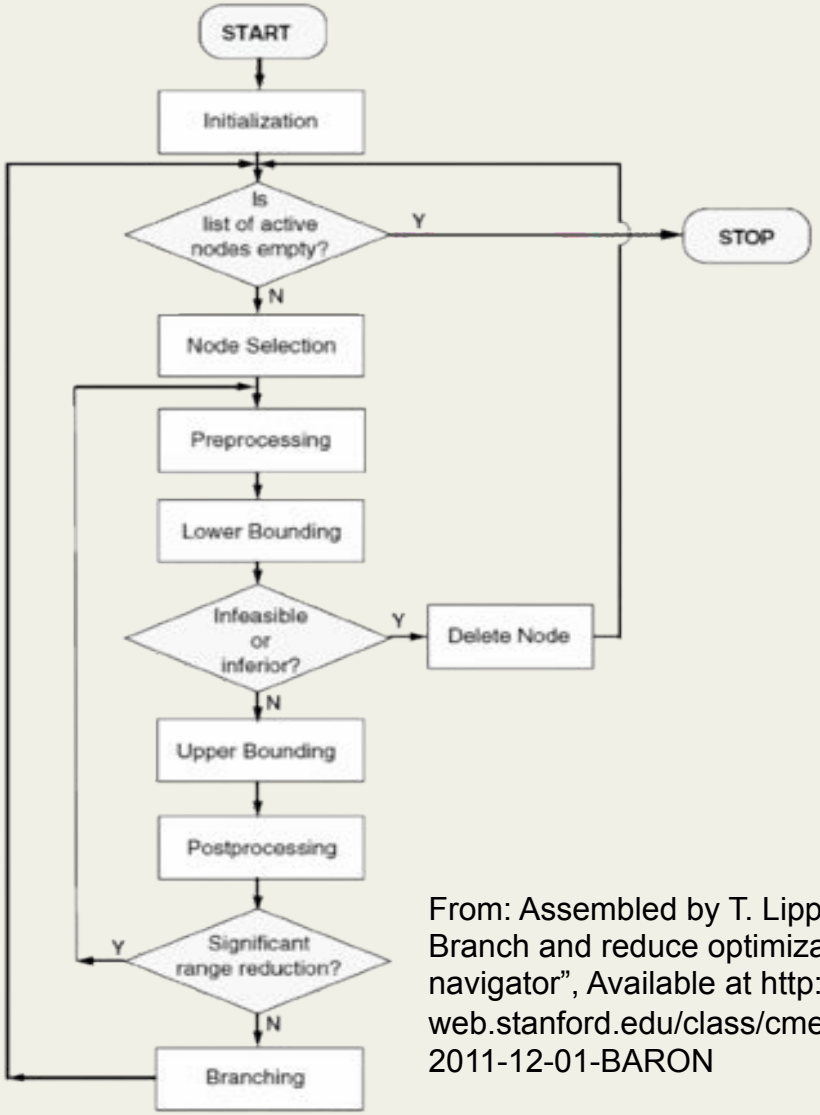
M. Tawarmalan and N. Sahinidis, "Global optimization of mixed-integer nonlinear programs: A theoretical and computational study", *Mathematical Programming,* vol. 99, no. 3, pp.563 -591, 2004.



Global Optimizer III

BARON

- Flow chart
- Up to 9-bus system
  - Time limit
  - Linear relaxations
  - Used for lower bound



From: Assembled by T. Lipp, "BARON: Branch and reduce optimization navigator", Available at http://web.stanford.edu/class/cme334/docs/2011-12-01-BARON

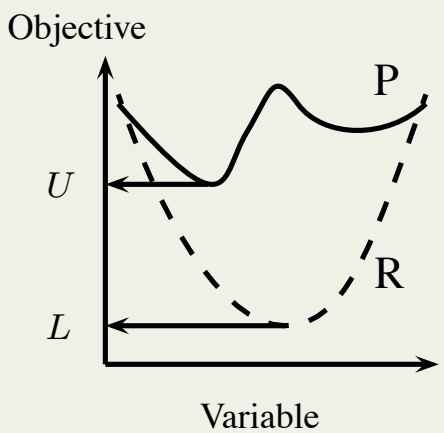
Global Optimizer IV

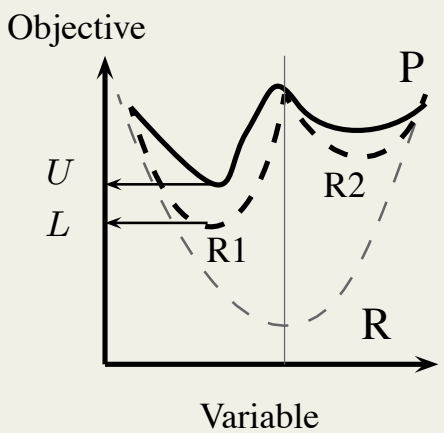
Branch-and-bound method (BB)

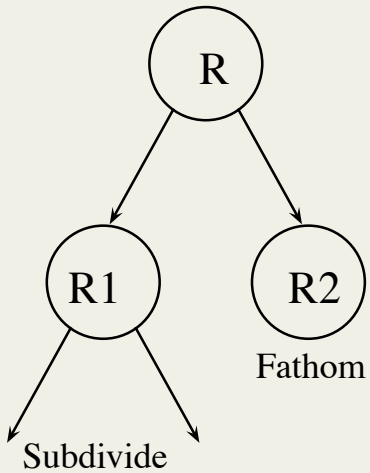
- At a given SDP solution, divide the feasible region with the control variables
- Within the feasible region, the global solution is sandwiched
  - Upper bound found by a local minimizer
  - Lower bound found by a convexified OPF
  - If upper bound and lower bound are close enough, the global solution in the region is found
- Search for all the regions and find the best solution
  → the global optimizer
- Observation:
  - BB finds the global optimizer
  - Finding the global solution ←→ Searching for the region that the global solution exists

Global Optimizer V

MITSUBISHI group implements BB for OPF







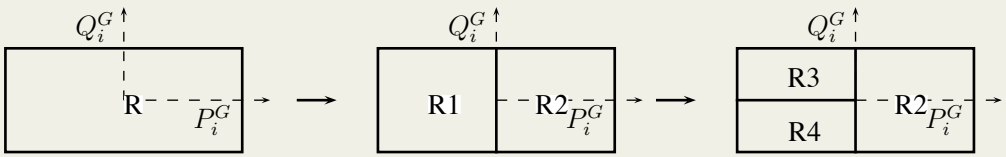
A. Gopalakrishnan, et. al., "Global optimization of optimal power flow using a branch and bound algorithm", in *Communication, Control, and Computing*, Allerton, pp. 609–616, 2012

- *U* set by CONOPT and *L* set by SDP
- Compare two *L*'s → keeps track of a "better" SDP

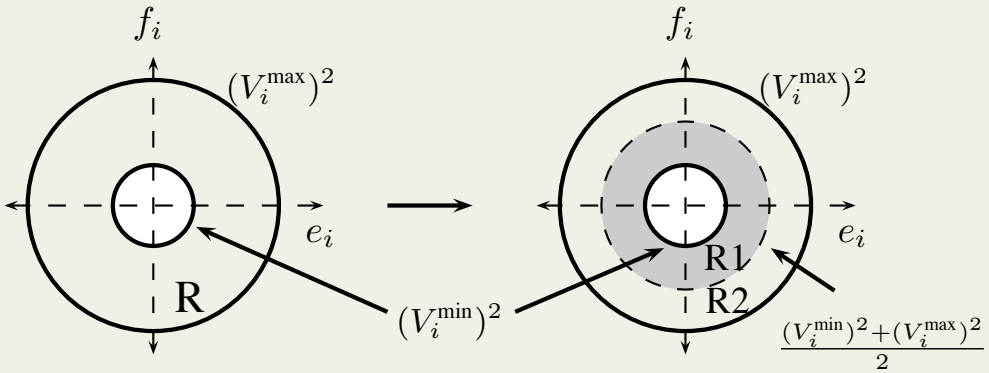
Global Optimizer VI

MITSUBISHI group implements BB for OPF

- Simple bisection of the feasible region
  - Voltage magnitude: radial bisections
  - Real power generation: rectangular bisections
  - Reactive power generation: rectangular bisections



(a) Rectangular Bisections

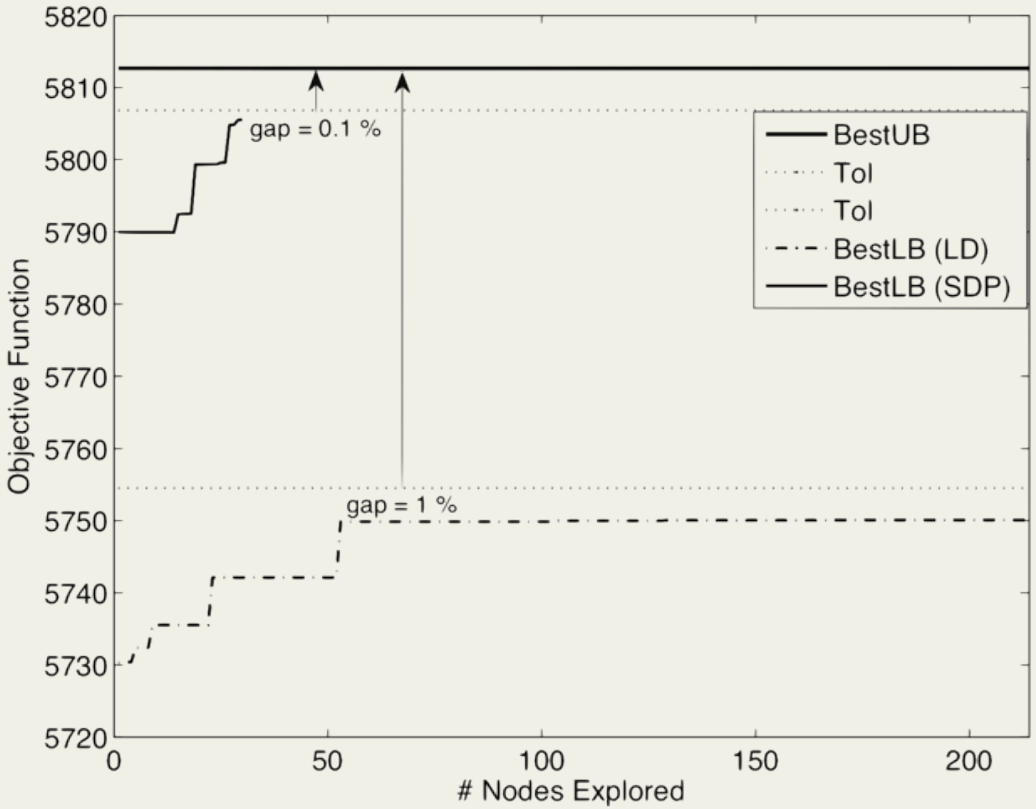


(b) Radial bisections

A. Gopalakrishnan, et. al., "Global optimization of optimal power flow using a branch and bound algorithm", in *Communication, Control, and Computing*, Allerton, pp. 609–616, 2012

Global Optimizer VII

- Branch-and-bound method
  - CONOPT solution is needed at every nodes
  - Initial CONOPT solution is the global optimum
  - 3-bus example: $\varepsilon_{obj} = 10^{-3}$
- Branch-and-reduce method
  - Does not reflect the complexity of OPF
  - Slow convergence at $\varepsilon_{obj} = 10^{-2}$



A. Gopalakrishnan, A. U. Raghunathan, D. Nikovski, and L. T. Biegler, "Global optimization of optimal power flow using a branch & bound algorithm," in Communication, Control, and Computing (Allerton), 2012 50th Annual Allerton Conference on. IEEE, 2012, pp. 609–616.

Global Optimizer VIII

MITSUBISHI group implements BB for OPF

- CONOPT and SDP to find upper and lower bounds
  - Even if SDP finds a rank-1 solution, the gap between two bounds can be nonzero because CONOPT find a local solution → needs to check feasibility of SDP solution
  - SDP solution is not exploited
  - Regardless SDP solution, bisect $|v|$, p, and q
- No bisection for voltage angle
- Objective function is convex on
  - Real power generation, and reactive power generation
    → p- and q- bisection may not be efficient

Research Idea I

Divide-and-conquer (DC)

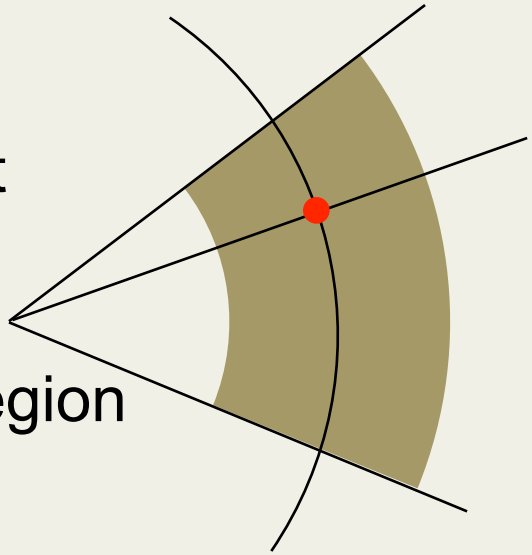
- It starts with a feasible solution (so-far-the-best, SFTB)
- Based on a given SDP solution, it divides the feasible region
- Within the feasible region, SDP finds a lower bound
  - If the lower bound ≥ SFTB, not worthwhile to explore
    → terminate the node
  - If the lower bound < SFTB
    - Rank-1 (feasible), update SFTB
    - Multiple rank solution, prune the node
  - If no nodes to explore, terminate the process and claim SFTB is the global solution

Research Idea II

Angular cut

- BB only considers the bisection of voltage magnitudes
- SDP finds a solution (red dot)
  - DC makes the circular cut and angular cut through the red dot

$$\underline{|v_i|} \le |v_i| \le \overline{|v_i|} \quad \text{and} \quad \underline{\theta_i} \le \theta_i \le \overline{\theta_i}$$



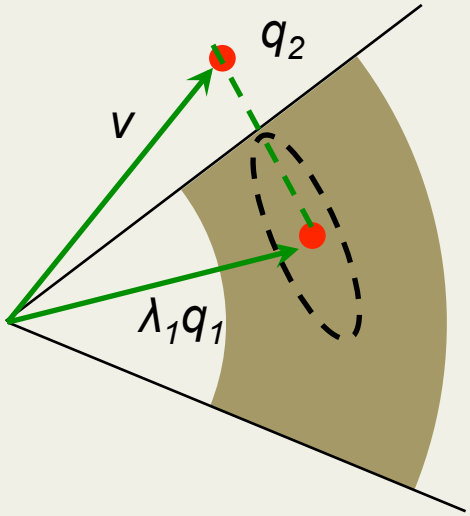
  - Efficient way to narrow down the region
  - Exploit the SDP solution

    → No revisit to infeasible *W*

Research Idea III

Meaning of multiple rank solution

- Primary eigenvector q_1 is an approximation of W
- Secondary eigenvector can be the maximum deviation that a "true" voltage exists from the primary eigenvector
- Largest element in the secondary eigenvector indicates the direction of the maximum deviation
- $W - \lambda_1^2 q_1 q_1^T \approx \lambda_2^2 q_2 q_2^T$: rank-1 approximation



Research Idea IV

- Need for a sub-optimization problem
  - SDP solution may not be rank-1
  - Eigenvalue decomposition of $W \approx \lambda^2 qq^T$, but λq is not inside the feasible region
  - We need an indicator (red dot) to divide the feasible region
- Sub-optimization problem
  - Need a vector v to approximate W
  - The vector must be inside the feasible region
  - Looking for a local solution

$$\left.\begin{array}{l} \min\limits_{v} \left\| vv^T - W \right\|_F \\ s.t. \left|\underline{v}\right| \le \left|v\right| \le \left|\overline{v}\right| \text{ and } \underline{\theta} \le \theta \le \overline{\theta} \end{array}\right\} \rightarrow v$$

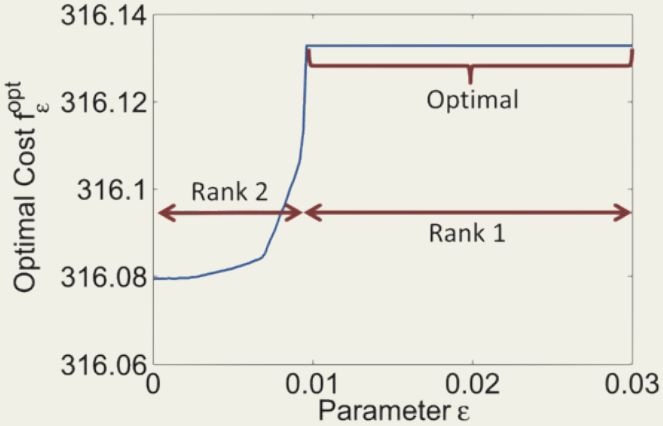
  - No need for EVD

Our Proposed Method I

- Checking the feasibility of the SDP solution
  - Eigenvalue decomposition in $\vartheta(N^3)$
  - $\lambda_2^2/\lambda_1^2 \neq 0$ → when to ignore the second eigenvalue?
  - Solve the sub-optimization problem → v
  - Power balance equations
  - Inequality constraints
  - If v is infeasible, check $W - vv^T$
- Comparison between the solution from SDP and SFTB
  - Terminate the node that is not worthwhile to explore
  - Terminate the node if SDP solution is feasible

Our Proposed Method II

- Two thresholds due to numerical errors
  - Checking feasibility of the SDP solution
  - Comparison between SDP solution and SFTB
    - If SDP finds a multiple rank solution and the relative difference ≤ ε_{obj}, then terminate the node
    - MITSUBISHI group choose $\varepsilon_{obj} = 10^{-3}$
- Penalized SDP relaxation: extra payment to *q* at *ε*
  - SDP solution becomes rank-1
  - System cost increases ~2×10^{-4}
  - As *ε* increases, wants to reduce *q* generation → low voltage sol.
    - MATPOWER also find the same sol.
    - Low-rank sol. costs $!

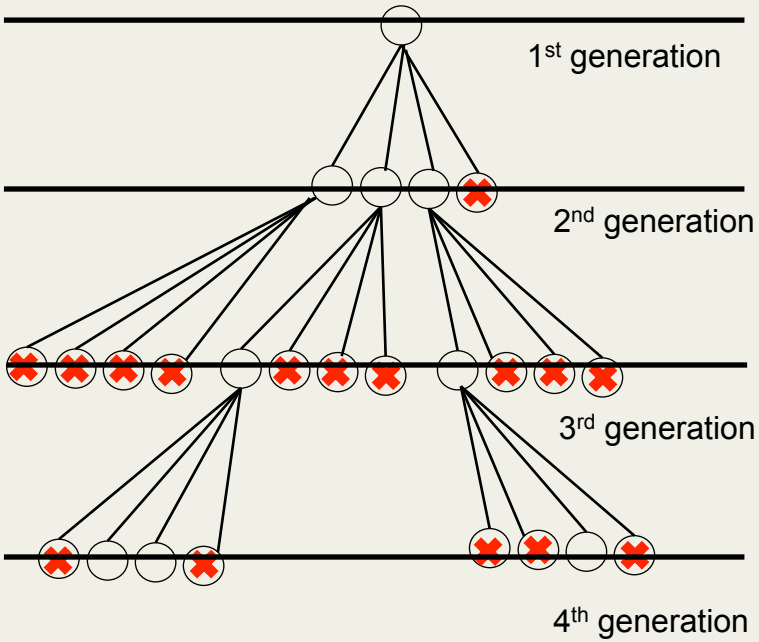


R. Madani, et. al. "Convex relaxation for optimal power flow problem: Mesh networks", *In Proc. Asilomar Conference on Signals, Systems and Computers*, November 2013.

Our Proposed Method III

Divide-and-conquer

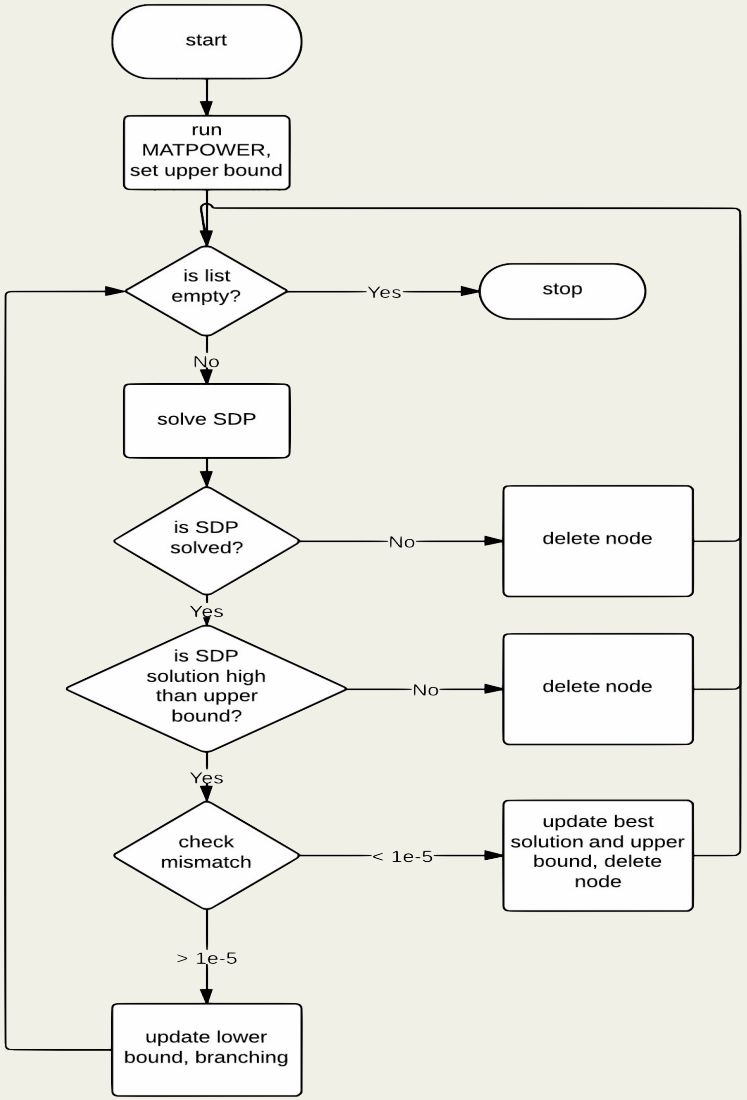
- MATPOWER → $SFTB_0$
- Criteria to terminate a node
  - Feasible SDP solution
  - SDP ≥ SFTB × $(1-\varepsilon_{obj})$
- Criteria to check feasibility
  - v from the sub-problem
  - $||vv^T - W||_F \leq \varepsilon_F$
  - Violation of constraints ≤ ε_{vio}
- Breadth first search
  - Active node → prune



Our Proposed Method IV

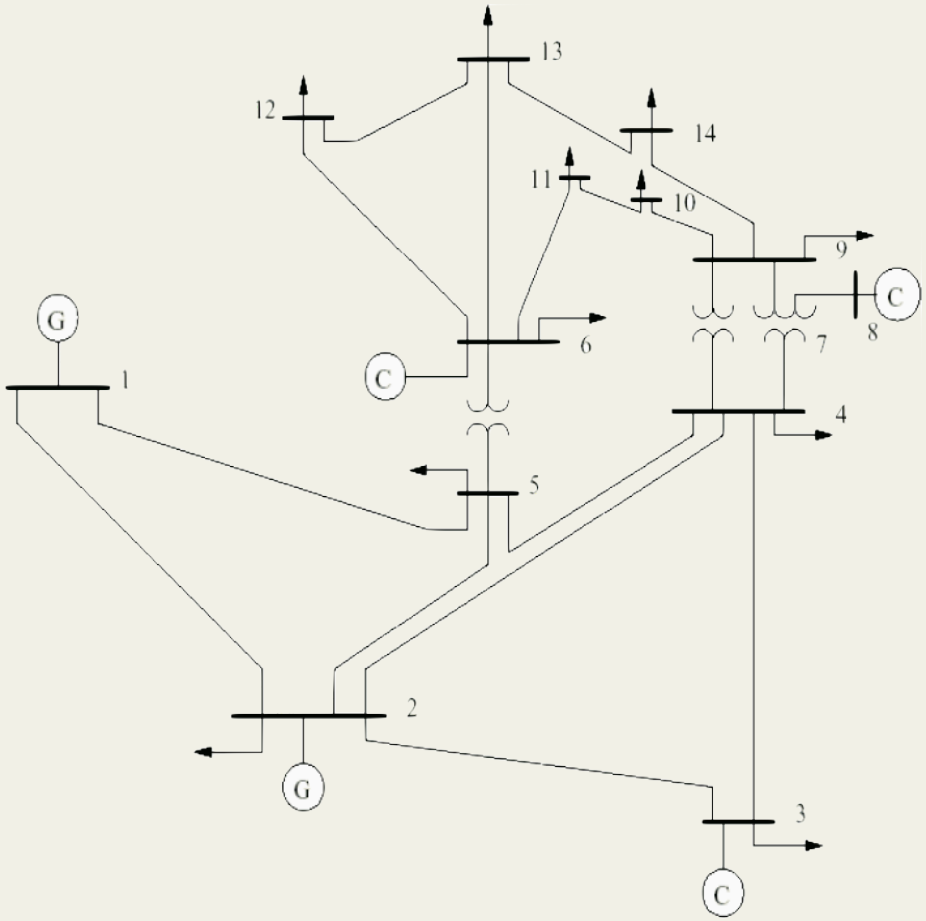
Divide-and-conquer

- Criteria to terminate a node
  - Feasible SDP solution
  - SDP ≥ SFTB × $(1-\varepsilon_{obj})$
- Criteria to check feasibility
  - v from the sub-problem
  - $||vv^T - W||_F \leq \varepsilon_F$
  - Mismatch from constraints $\leq \varepsilon_{mis}$



Simulation Environment

- IEEE 14-bus system
  - 5 generators
  - 20 lines
- MATPOWER finds
  - System cost: $3091.4 ← initial SFTB
- DC OPF
  - System cost: $3048.8
- SDP finds a rank-3 solution
  - Physically not meaningful
  - System cost: $3077.1 ← lower bound for the global solution



MATPOWER Solution

- System cost = $3091.4/h
- Real power loss = 2.739MW
- Violation $\Delta = \begin{bmatrix} |g_{eq}| \\ \max(g_{ineq}, 0) \end{bmatrix}$

  where $g_{eq} = 0;\ g_{ineq} \le 0$

- $||\Delta||_2 = 1.5\times10^{-8}$

$$|v| = \begin{bmatrix} 1.050 \\ 1.042 \\ 1.027 \\ 1.018 \\ 1.026 \\ 1.016 \\ 0.979 \\ 0.966 \\ 0.967 \\ 0.967 \\ 0.987 \\ 0.998 \\ 0.990 \\ 0.958 \end{bmatrix};\ \theta / rad = \begin{bmatrix} 0.000 \\ -0.025 \\ -0.074 \\ -0.057 \\ -0.045 \\ -0.038 \\ -0.047 \\ 0.014 \\ -0.080 \\ -0.077 \\ -0.060 \\ -0.056 \\ -0.059 \\ -0.091 \end{bmatrix};\ \begin{cases} \overline{|v|}_2 = 1.050 \\ \underline{|v|}_2 = 0.958 \end{cases}$$

$$p_G / MW = \begin{bmatrix} 68.808 \\ 40.000 \\ 60.000 \\ 60.000 \\ 32.931 \end{bmatrix}$$

$$q_G / MVA = \begin{bmatrix} 0.003 \\ 19.492 \\ 23.814 \\ 24.000 \\ -5.787 \end{bmatrix}$$

DC OPF Solution

- System cost = $3048.8/h

$$\theta / rad = \begin{bmatrix} 0.000 \\ -0.033 \\ -0.105 \\ -0.074 \\ -0.058 \\ -0.044 \\ -0.059 \\ 0.006 \\ -0.092 \\ -0.091 \\ -0.071 \\ -0.066 \\ -0.071 \\ -0.106 \end{bmatrix}; p_G / MW = \begin{bmatrix} 81.830 \\ 40.000 \\ 40.000 \\ 60.000 \\ 37.170 \end{bmatrix} \rightarrow \frac{\left\| p_G^{DC} - p_G^{MATPOWER} \right\|_2}{\left\| p_G^{MATPOWER} \right\|_2} = 0.20$$

SDP Solution

- System cost = \$3077.1/h (\$3091.4/h from MATPOWER)
- Rank(W) = 3, $||\Delta||_2 = 0.12$

$$W = EDE^T$$

$$\rightarrow \lambda^2 = \begin{pmatrix} 14.405 \\ 0.050 \\ 0.022 \end{pmatrix} \; ; \; |v|_{EVD} = \begin{bmatrix} 1.049 \\ 1.041 \\ 1.029 \\ 1.025 \\ 1.030 \\ 1.022 \\ 1.008 \\ 1.046 \\ 0.986 \\ 0.984 \\ 0.999 \\ 1.006 \\ 0.998 \\ 0.972 \end{bmatrix} ; \; \theta_{EVD} / rad = \begin{bmatrix} 0.000 \\ -0.021 \\ -0.069 \\ -0.051 \\ -0.040 \\ -0.028 \\ -0.031 \\ 0.037 \\ -0.069 \\ -0.067 \\ -0.050 \\ -0.046 \\ -0.050 \\ -0.080 \end{bmatrix}$$

$$\rightarrow W \approx \lambda_1^2 u_1 u_1^T$$

$$\rightarrow v = \lambda_1 u_1$$

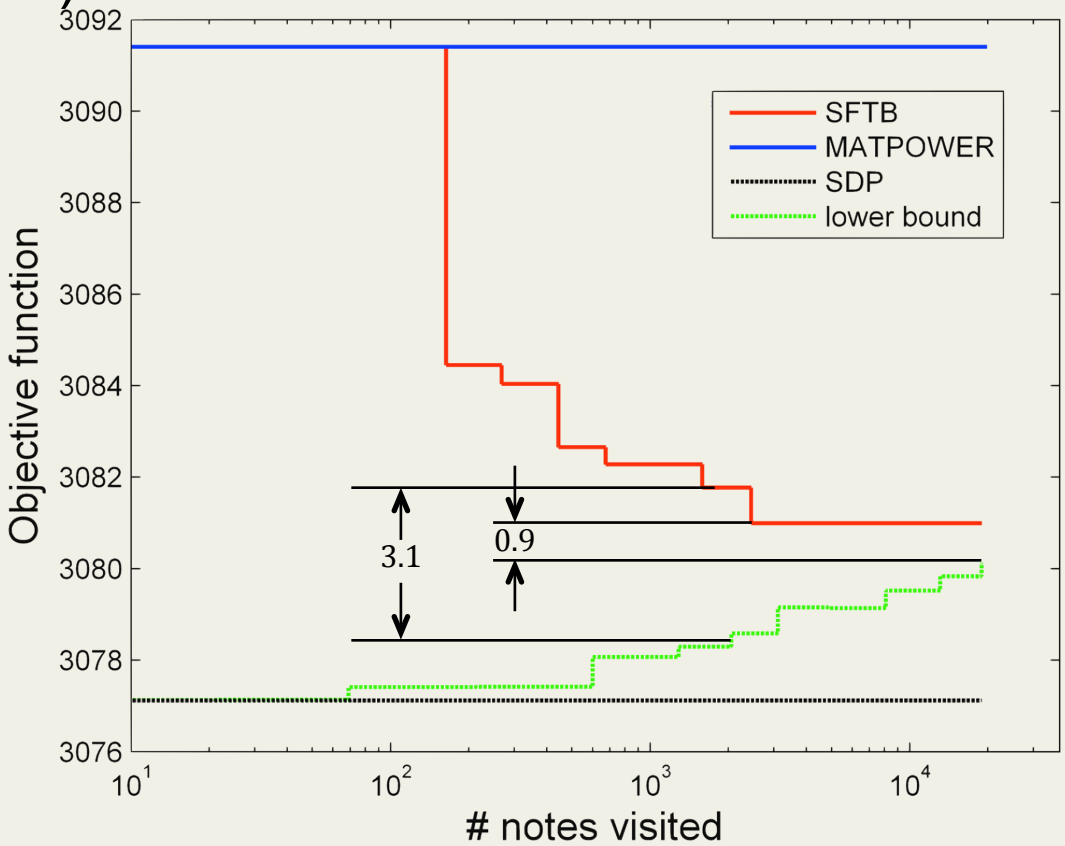
$$\begin{cases} \overline{|v|_2} = 1.049 \\ \underline{|v|_2} = 0.972 \end{cases}$$

$$p_G / MW = \begin{bmatrix} 60.108 \\ 40.000 \\ 60.000 \\ 60.000 \\ 41.115 \end{bmatrix}$$

$$q_G / MVA = \begin{bmatrix} 0.000 \\ 9.713 \\ 21.808 \\ 24.000 \\ 24.000 \end{bmatrix}$$

Performance of Our Algorithm I

- DC method
  - Early termination based on the quality of multiple rank sol.
  - Visited 20,000 nodes to find the global solution
  - $||\Delta||_2 = 2.3\times10^{-6}$ (0.12 for SDP)
  - $\varepsilon_{obj} = 3\times10^{-4}$ (10^{-3} for BB)
- Problems observed
  - High cost of SDP for a large system
  - Sub-optimization problem: most time consuming
  - Many nodes to visit



Performance of Our Algorithm II

Divide-and-conquer method

- System cost in $/h
  - This study = 3081.0
  - MATPOWER = 3091.4
  - SDP = 3077.1
- Lowest voltage magnitude
  - This study = 0.983
  - MATPOWER = 0.958
  - SDP = 0.972
- Real power loss in MW
  - This study = 2.309
  - MATPOWER = 2.739
  - SDP = 2.223

$$|v| = \begin{bmatrix} 1.050 \\ 1.041 \\ 1.028 \\ 1.029 \\ 1.034 \\ 1.033 \\ 1.016 \\ 1.037 \\ 0.997 \\ 0.995 \\ 1.010 \\ 1.017 \\ 1.009 \\ 0.983 \end{bmatrix}; \quad \theta / rad = \begin{bmatrix} 0.000 \\ -0.022 \\ -0.071 \\ -0.056 \\ -0.044 \\ -0.033 \\ -0.041 \\ 0.021 \\ -0.074 \\ -0.072 \\ -0.055 \\ -0.051 \\ -0.054 \\ -0.084 \end{bmatrix}; \quad \begin{cases} \overline{|v|_2} = 1.050 \\ \underline{|v|_2} = 0.983 \end{cases}$$

$$p_G / MW = \begin{bmatrix} 63.980 \\ 40.000 \\ 60.000 \\ 60.000 \\ 37.329 \end{bmatrix} \quad q_G / MVA = \begin{bmatrix} 0.000 \\ 3.127 \\ 19.117 \\ 23.801 \\ 13.978 \end{bmatrix}$$

Conclusions

- Our DC finds the global solution in an efficient way by
  - Dividing regions with voltage cut and angular cut
  - Finding the ideal place to prune using the sub-optimization problem
  - Terminating a node efficiently
- Issues identified with high computation cost regarding
  - Sub-optimization problem
  - SDP with a large system
  - Many nodes to visit