Efficient AC Optimal Power Flow & Global Optimizer Solutions

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Overview

- Optimal power flow
- Polar vs. Cartesian coordinate system
- Global optimizer
- Research idea
- Our proposed method
- Simulation result
- Conclusions
Optimal Power Flow I

- Finds an optimal solution while satisfying
  - Kirchhoff’s laws
  - Operation constraints
  - Economic, policy, and/or environmental constraints
- Plays a key role in operation and planning of
  - Smart grid technologies
  - Renewable energy integration
- Nonlinear and nonconvex \( \rightarrow \) difficult to solve
  - No guarantee to find a solution
  - Heuristic search aiming for a local solution
Optimal Power Flow II

OPF in the polar coordinate system

- **Control variables**
  - Voltage magnitude \((E)\) and angle \((\theta)\)
  - Real \((p)\) and reactive \((q)\) power generation

\[
\min _{E,\theta,p,q} \sum _{i=1}^{N_g} \left[ Cost_p^i (p_i) + Cost_q^i (q_i) \right]
\]

\[
\begin{cases}
g_p (E, \theta, p) = 0; & g_q (E, \theta, q) = 0 \\
flow (E, \theta) \leq 0; & E \leq E \leq \bar{E} \\
p \leq p \leq \bar{p}; & q \leq q \leq \bar{q}
\end{cases}
\]

- **MATPOWER** finds a local solution
  - Often observes \(E \sim 1.0\) and \(-\pi/3 \leq \theta \leq \pi/3\)
Polar vs. Cartesian Coordinate System I

- Power balance equations are sinusoidal function with $\theta$
- 3-bus system:
  - Bus 1: reference bus
  - Bus 2: PV bus
- Power flow solutions at various $|v|$ and power generation at Bus 2
- Power generation is a function of $|v|$ and $\theta$ at Bus 2
  - $|v|$ and $\theta$ are independent
  - Efficient search in the space of $|v|$ and $\theta$ is possible
Optimal Power Flow III

OPF in the Cartesian coordinate system

- Control variables
  - Real \((x)\) and imaginary \((y)\) components of voltages
  - Real \((p)\) and reactive \((q)\) power generation

\[
\min_{x,y,p,q} \sum_{i=1}^{N_g} [\text{Cost}_p^i (p_i) + \text{Cost}_q^i (q_i)] \\
\begin{cases} \\
g_p(x, y, p) = 0; \quad g_q(x, y, q) = 0 \\
\text{flow}(x, y) \leq 0; \quad E^2 \leq E(x, y) \leq \bar{E}^2 \\
p \leq p \leq \bar{p}; \quad q \leq q \leq \bar{q} \\
\end{cases}
\]

- \(E\) and \(g\) are quadratic functions in \(x\) and \(y\)
- \(\text{flow}\) is a quartet function in \(x\) and \(y\)
Polar vs. Cartesian Coordinate System II

- Power balance equations are quadratic in $x$ and $y$
- $x$ and $y$ are dependent
- Saddle points?
- Different strategy for search
Optimal Power Flow IV

- Semi-definite programming (SDP) relaxes non-convexity

\[
\min_{W, p, q} \sum_{i=1}^{N_c} \left[ \text{Cost}_p^i (p_i) + \text{Cost}_q^i (q_i) \right]
\]

\[
\begin{align*}
\text{s.t. } & \quad p_g^i - p_d^i = \text{Tr} (\Phi_p^i W) ; 
\quad q_g^i - q_d^i = \text{Tr} (\Phi_q^i W) ; \\
& \quad E_i^2 \leq \text{Tr} (\Pi_i W) \leq \bar{E}_i^2 ; 
\quad \underline{p} \leq p \leq \bar{p} ; 
\quad \underline{q} \leq q \leq \bar{q} ; 
\quad W \succeq 0
\end{align*}
\]

\[
\begin{bmatrix}
-\text{flow}_i^2 
\text{Tr}(Y_i W) 
\text{Tr}(\tilde{Y}_i W)
\end{bmatrix}
\preceq 0
\]

- \( W = [x^T \ y^T]^T [x^T \ y^T] \)
- \( \text{Rank}(W) = 1 \) is relaxed with \( W \succeq 0 \)
- If solution \( W \) is rank-1 \( \rightarrow \) global solution
- If not
  - In other application areas \( W \approx \lambda^2 qq^T \)
  - In power systems, no physically meaningful solution
- SDP yields a lower bound of all local minimizers
Global Optimizer I

Algorithms seeking for the global optimizer

- Stochastic methods
  - Simulated annealing
  - Direct Monte-Carlo sampling
  - Stochastic tunneling
- Deterministic methods
  - Cutting plane method, and
  - Branch-and-bound method
- Our goal is to guarantee to find the global optimizer if exists
Global Optimizer II

Branch-and-reduce (BR)

- BARON is a widely used commercial software package to find the global solution
- OPF in the polar coordinate system cannot be solved with BARON because it does not take sinusoidal functions
- Convexification through a linear approximation
- Rule for selecting supporting lines in sandwich algorithm
  - Maximum error rule
  - Two sub-regions

Global Optimizer III

BARON
- Flow chart
- Up to 9-bus system
  - Time limit
  - Linear relaxations
  - Used for lower bound

Global Optimizer IV

Branch-and-bound method (BB)

- At a given SDP solution, divide the feasible region with the control variables
- Within the feasible region, the global solution is sandwiched
  - Upper bound found by a local minimizer
  - Lower bound found by a convexified OPF
  - If upper bound and lower bound are close enough, the global solution in the region is found
- Search for all the regions and find the best solution
  → the global optimizer
- Observation:
  - BB finds the global optimizer
  - Finding the global solution ↔ Searching for the region that the global solution exists
Global Optimizer V

MITSUBISHI group implements BB for OPF

- $U$ set by CONOPT and $L$ set by SDP
- Compare two $L$’s $\rightarrow$ keeps track of a “better” SDP

A. Gopalakrishnan, et. al., “Global optimization of optimal power flow using a branch and bound algorithm”, in Communication, Control, and Computing, Allerton, pp. 609–616, 2012
Global Optimizer VI

MITSUBISHI group implements BB for OPF

- Simple bisection of the feasible region
  - Voltage magnitude: radial bisections
  - Real power generation: rectangular bisections
  - Reactive power generation: rectangular bisections

\[ \begin{align*}
Q_{i}^{G} & \rightarrow R_{i} - \bar{p}_{i}^{G} \\
Q_{i}^{G} & \rightarrow R1 - R2 + \bar{p}_{i}^{G} \\
Q_{i}^{G} & \rightarrow R3 - R2 + \bar{p}_{i}^{G}
\end{align*} \]

(a) Rectangular Bisections

\[ \begin{align*}
& (V_{i}^{\text{max}})^{2} \\
& (V_{i}^{\text{min}})^{2} \\
& (V_{i}^{\text{min}})^{2} + (V_{i}^{\text{max}})^{2} / 2
\end{align*} \]

(b) Radial bisections

A. Gopalakrishnan, et. al., “Global optimization of optimal power flow using a branch and bound algorithm”, in Communication, Control, and Computing, Allerton, pp. 609–616, 2012
Global Optimizer VII

- Branch-and-bound method
  - CONOPT solution is needed at every node
  - Initial CONOPT solution is the global optimum
  - 3-bus example: $\varepsilon_{\text{obj}} = 10^{-3}$
- Branch-and-reduce method
  - Does not reflect the complexity of OPF
  - Slow convergence at $\varepsilon_{\text{obj}} = 10^{-2}$

Global Optimizer VIII

MITSUBISHI group implements BB for OPF

- CONOPT and SDP to find upper and lower bounds
  - Even if SDP finds a rank-1 solution, the gap between two bounds can be nonzero because CONOPT find a local solution \( \rightarrow \) needs to check feasibility of SDP solution
  - SDP solution is not exploited
  - Regardless SDP solution, bisect \(|v|, p, \text{ and } q\)
- No bisection for voltage angle
- Objective function is convex on
  - Real power generation, and reactive power generation \( \rightarrow p\)- and \(q\)- bisection may not be efficient
Research Idea I

Divide-and-conquer (DC)

- It starts with a feasible solution (so-far-the-best, SFTB)
- Based on a given SDP solution, it divides the feasible region
- Within the feasible region, SDP finds a lower bound
  - If the lower bound $\geq$ SFTB, not worthwhile to explore
    $\rightarrow$ terminate the node
  - If the lower bound $<$ SFTB
    - Rank-1 (feasible), update SFTB
    - Multiple rank solution, prune the node
  - If no nodes to explore, terminate the process and claim SFTB is the global solution
Research Idea II

Angular cut

- BB only considers the bisection of voltage magnitudes
- SDP finds a solution (red dot)
  - DC makes the circular cut and angular cut through the red dot
  \[ |v_i| \leq |v_i| \leq |v_i| \text{ and } \theta_i \leq \theta_i \leq \theta_i \]
- Efficient way to narrow down the region
- Exploit the SDP solution
  \[ \rightarrow \text{No revisit to infeasible } W \]
Research Idea III

Meaning of multiple rank solution

- Primary eigenvector $q_1$ is an approximation of $W$
- Secondary eigenvector can be the maximum deviation that a “true” voltage exists from the primary eigenvector
- Largest element in the secondary eigenvector indicates the direction of the maximum deviation
- $W - \lambda_1^2 q_1 q_1^T \approx \lambda_2^2 q_2 q_2^T$: rank-1 approximation
Research Idea IV

- Need for a sub-optimization problem
  - SDP solution may not be rank-1
  - Eigenvalue decomposition of $W \approx \lambda^2 qq^T$, but $\lambda q$ is not inside the feasible region
  - We need an indicator (red dot) to divide the feasible region
- Sub-optimization problem
  - Need a vector $v$ to approximate $W$
  - The vector must be inside the feasible region
  - Looking for a local solution
    \[
    \min_{v} \| vv^T - W \|_F \\
    \text{s.t.} \quad |v| \leq |\bar{v}| \quad \text{and} \quad \theta \leq \theta \leq \bar{\theta}
    \] 
    \rightarrow v
  - No need for EVD
Our Proposed Method I

- Checking the feasibility of the SDP solution
  - Eigenvalue decomposition in $\mathcal{O}(N^3)$
  - $\lambda_2^2/\lambda_1^2 \neq 0 \Rightarrow$ when to ignore the second eigenvalue?
  - Solve the sub-optimization problem $\Rightarrow \nu$
  - Power balance equations
  - Inequality constraints
  - If $\nu$ is infeasible, check $W - \nu\nu^T$
- Comparison between the solution from SDP and SFTB
  - Terminate the node that is not worthwhile to explore
  - Terminate the node if SDP solution is feasible
Our Proposed Method II

- Two thresholds due to numerical errors
  - Checking feasibility of the SDP solution
  - Comparison between SDP solution and SFTB
    - If SDP finds a multiple rank solution and the relative difference \( \leq \epsilon_{\text{obj}} \), then terminate the node
    - MITSUBISHI group choose \( \epsilon_{\text{obj}} = 10^{-3} \)
- Penalized SDP relaxation: extra payment to \( q \) at \( \epsilon \)
  - SDP solution becomes rank-1
  - System cost increases \( \sim 2 \times 10^{-4} \)
  - As \( \epsilon \) increases, wants to reduce \( q \) generation \( \rightarrow \) low voltage sol.
  - MATPOWER also find the same sol.
  - Low-rank sol. costs $!

Our Proposed Method III

Divide-and-conquer

- **MATPOWER → SFTB₀**
- Criteria to terminate a node
  - Feasible SDP solution
  - \( SDP ≥ SFTB × (1-\varepsilon_{\text{obj}}) \)
- Criteria to check feasibility
  - \( v \) from the sub-problem
  - \( ||vv^T - W||_F ≤ \varepsilon_F \)
  - Violation of constraints ≤ \( \varepsilon_{\text{vio}} \)
- Breadth first search
  - Active node → prune
Our Proposed Method IV

Divide-and-conquer

- Criteria to terminate a node
  - Feasible SDP solution
  - SDP ≥ SFTB \times (1-\varepsilon_{\text{obj}})

- Criteria to check feasibility
  - $v$ from the sub-problem
  - $||vv^T - W||_F \leq \varepsilon_F$
  - Mismatch from constraints $\leq \varepsilon_{\text{mis}}$
Simulation Environment

- IEEE 14-bus system
  - 5 generators
  - 20 lines
- MATPOWER finds
  - System cost: $3091.4 $\leftarrow$ initial SFTB
- DC OPF
  - System cost: $3048.8
- SDP finds a rank-3 solution
  - Physically not meaningful
  - System cost: $3077.1 $\leftarrow$ lower bound for the global solution
MATPOWER Solution

- System cost = $3091.4/h
- Real power loss = 2.739MW
- Violation \( \Delta = \begin{bmatrix} \left| g_{eq} \right| \\ \max \left( g_{ineq}, 0 \right) \end{bmatrix} \)
  
  where \( g_{eq} = 0; \ g_{ineq} \leq 0 \)
  
  - \( \| \Delta \|_2 = 1.5 \times 10^{-8} \)

\[
\begin{bmatrix}
1.050 \\
1.042 \\
1.027 \\
1.018 \\
1.026 \\
1.016 \\
0.979 \\
0.966 \\
0.967 \\
0.967 \\
0.987 \\
0.998 \\
0.990 \\
0.958 \\
\end{bmatrix}
\begin{bmatrix}
0.000 \\
-0.025 \\
-0.074 \\
-0.057 \\
-0.045 \\
-0.038 \\
0.047 \\
0.014 \\
-0.080 \\
-0.077 \\
-0.060 \\
-0.056 \\
-0.059 \\
-0.091 \\
\end{bmatrix}
\]

\[
\begin{aligned}
|v| &= 0.979 \\
\theta / \text{rad} &= 0.000 \\
\end{aligned}
\]

\[
\begin{bmatrix}
|v|_2 = 1.050 \\
|v|_2 = 0.958 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
68.808 \\
40.000 \\
60.000 \\
32.931 \\
0.003 \\
19.492 \\
23.814 \\
24.000 \\
-5.787 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
p_G / \text{MW} \\
q_G / \text{MVA} \\
\end{bmatrix}
\]
DC OPF Solution

- System cost = $3048.8/h

\[
\theta / \text{rad} = \begin{bmatrix} 0.000 \\ -0.033 \\ -0.105 \\ -0.074 \\ -0.058 \\ -0.044 \\ -0.059 \\ 0.006 \\ -0.092 \\ -0.091 \\ -0.071 \\ -0.066 \\ -0.071 \\ -0.106 \end{bmatrix};
\]

\[
p_G / \text{MW} = \begin{bmatrix} 81.830 \\ 40.000 \\ 40.000 \\ 60.000 \\ 37.170 \end{bmatrix}
\]

\[
\left\| P_G^{DC} - P_G^{\text{MATPOWER}} \right\|_2 = 0.20
\]
SDP Solution

- System cost = $3077.1/h ($3091.4/h from MATPOWER)
- Rank(W) = 3, \( \| \Delta \|_2 = 0.12 \)

\[
W = EDE^T
\]

\[
\lambda^2 = \begin{pmatrix} 14.405 \\ 0.050 \\ 0.022 \end{pmatrix}, \quad \| v \|_{EV} = \begin{bmatrix} 1.049 \\ 1.041 \\ 1.029 \\ 1.025 \\ 1.030 \\ 1.022 \\ 1.008 \\ 1.046 \end{bmatrix}, \quad \| v \|_{EV} = \begin{bmatrix} 0.000 \\ -0.021 \\ -0.069 \\ -0.051 \\ -0.040 \\ -0.028 \\ -0.031 \\ 0.037 \end{bmatrix}
\]

\[
\| v \|_2 = 1.049, \quad \| v \|_2 = 0.972
\]

\[
p_G/MW = \begin{bmatrix} 60.108 \\ 40.000 \end{bmatrix}, \quad q_G/MVA = \begin{bmatrix} 0.000 \\ 9.713 \\ 21.808 \\ 24.000 \\ 24.000 \end{bmatrix}
\]
Performance of Our Algorithm I

- DC method
  - Early termination based on the quality of multiple rank sol.
  - Visited 20,000 nodes to find the global solution
  - $||\Delta||_2 = 2.3 \times 10^{-6}$ (0.12 for SDP)
  - $\varepsilon_{obj} = 3 \times 10^{-4}$ ($10^{-3}$ for BB)
- Problems observed
  - High cost of SDP for a large system
  - Sub-optimization problem: most time consuming
  - Many nodes to visit
Performance of Our Algorithm II

Divide-and-conquer method

- System cost in $/h
  - This study = 3081.0
  - MATPOWER = 3091.4
  - SDP = 3077.1

- Lowest voltage magnitude
  - This study = 0.983
  - MATPOWER = 0.958
  - SDP = 0.972

- Real power loss in MW
  - This study = 2.309
  - MATPOWER = 2.739
  - SDP = 2.223

\[
|v| = \begin{bmatrix} 1.050 \\ 1.041 \\ 1.028 \\ 1.029 \\ 1.034 \\ 1.033 \\ 1.016 \\ 1.037 \\ 0.997 \\ 0.995 \\ 1.010 \\ 1.017 \\ 1.009 \\ 0.983 \end{bmatrix} \quad \theta / \text{rad} = \begin{bmatrix} 0.000 \\ -0.022 \\ -0.071 \\ -0.056 \\ -0.044 \\ -0.033 \\ -0.041 \\ -0.021 \\ -0.074 \end{bmatrix}
\]

\[
\begin{bmatrix} |v|_2 \end{bmatrix} = 1.050 \quad \begin{bmatrix} \theta / \text{rad} \end{bmatrix} = \begin{bmatrix} 63.980 \\ 40.000 \end{bmatrix}
\]

\[
p_G / \text{MW} = \begin{bmatrix} 60.000 \\ 37.329 \end{bmatrix} \quad q_G / \text{MVA} = \begin{bmatrix} 0.000 \\ 3.127 \end{bmatrix}
\]
Conclusions

• Our DC finds the global solution in an efficient way by
  • Dividing regions with voltage cut and angular cut
  • Finding the ideal place to prune using the sub-optimization problem
  • Terminating a node efficiently
• Issues identified with high computation cost regarding
  • Sub-optimization problem
  • SDP with a large system
  • Many nodes to visit