Random Topology Power Grid Modeling and Automated Simulation Platform

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Project Objectives

- Complete a comprehensive study of realistic power grid topology and parameters.
- Formulate appropriate random topology power grid models.
- Provide an automated simulation platform
  - enable sufficient testing and verification of new concepts, methods, and controls in power grids
  - compatible with MATPOWER
Motivation of the Project

Future power engineers and researchers need appropriate randomly generated grid network topologies to do genuine Monte Carlo experiments.

- If the random networks are truly representative and if the concepts or methods test well in this environment they would test well on any instance of such a network.

Current situation

- extremely difficult to obtain realistic system data from utilities
- limited reference test cases
- existing models with shortcomings, i.e. connectivity and scaling property
Critical Applications for the Grid

• Renewable generation interconnection
• PMU placements to facilitate fast state estimation and real-time state awareness
• Transmission expansion planning
• Grid vulnerability and security analysis
• Transient stability controls
• Power market strategy experiments
• Smart grid communication infrastructure
  – especially when the communication network layout partially or fully overlaps with the grid itself, e.g. power line communication.
Precursors of our study

• Most of the literature has used one specific real grid or reference models for testing
  – IEEE 30 57, 118 and 300 bus systems
  – Power systems test case archive: 
    http://www.ee.washington.edu/research/pstca/

• Scalable models to grasp macroscopic trends
  – [Parashar and Thorp ’04] ring topology
  – [Rosas-Casals, Valverde, Solé ’07] tree topology
  – [Watts, Strogatz ’98], small-world topology
  – Other models
• 3 sections in transmission
  – High, Medium and Low voltage sections
Admittance matrix and the graph topology

- **Line-Node Incidence Matrix** $A$ ($M \times N$):
  Line $m$: node $i$ – node $j$ → $A_{m,i} = 1, A_{m,j} = -1$
  $else$, $A_{m,k} = 0$.

- **Admittance matrix**
  \[
  Y = A^T \text{diag}(y_1, \ldots, y_M)A
  \]

- **Graph Laplacian**: $L = A^T A$

- **Observation**: $Y$ is a complex-weighted Laplacian

- **Complex weights given by the admittances of the lines**
  $y_l = 1/z_l = 1/(r_l + jx_l)$
The Laws for the Grid

- Voltage, Currents, Powers → narrow spectrum

Alternating Current (AC) ~ frequency $f_0$ 60 or 50 Hz

- “phasors” (complex numbers whose phase and amplitude match the AC signal $V$ or $I$), e.g.

$$ v(t) = \sqrt{2}|V| \exp(j\theta) \exp(j2\pi f_0 t) \quad \longrightarrow \quad V = |V| \exp(j\theta) $$

- Kirchhoff’s Voltage/Current laws (KVL-KCL)

$$ \sum_{i \in \text{Circuit}} V_i = 0, \quad \sum_{i \in \text{Node}} I_i = 0 $$

- Ohm’s law

$$ V_i = Z_i I_i $$
Statistical Modeling of Power Grid

- Topological and electrical characteristics of the transmission grid
- As captured by the statistical properties of the grid admittance matrix $Y$
- Propose a random topology power grid model with plausible topology and electrical parameters
Statistical Modeling of Power Grid

- Topology
- Electrical Parameters

Rand-topo Power Grid Model
Power Grid – Network Topology

- Small-world Properties
- Node Degree Distribution
- Connectivity Scaling
- etc
Power Grid – Electrical Parameters

- The line impedance distribution – heavy-tailed
Random topology models

• ‘98 Watts and Strogatz, *Nature*
  – *Conjecture: Power Grids are small world networks*

  Erdos Reny  Small World  Power network

– *Topological studies*: [Newman ’03], [Whitney & Alderson’06], [Wang, Rong,’09]

– *Degree distribution*: [Albert et al. ‘04], [Rosas-Casals et al. ‘07]
Small world: high clustering coefficient

- high average clustering coefficient of the sample power grid network examined

Definition of clustering coefficient

\[ C_i = \frac{2|\{e_{jk}\}|}{k_i(k_i - 1)} \forall (v_j, v_k) \in N_i, e_{jk} \in E \]

\[ (N_i = \text{neighbors of } v_i) \]

\[ (E = \text{edges}) \]

\[ (k_i = \text{degree of } i) \]

<table>
<thead>
<tr>
<th>Grid</th>
<th>( C(G) )</th>
<th>( C(R) )</th>
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<tbody>
<tr>
<td>IEEE-30</td>
<td>0.2348</td>
<td>0.094253</td>
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<td>0.0856</td>
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<tr>
<td>WSCC-4941</td>
<td>0.0801</td>
<td>0.000540</td>
</tr>
</tbody>
</table>
Small world: short average path length

• Observation: $\langle l \rangle \approx 3 \log(N)$

<table>
<thead>
<tr>
<th>Topological Characteristics of Real-World Power Networks</th>
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<tr>
<td>$(N, m)$</td>
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</table>

N: Number of nodes  
m: number of lines  
$<k>$ Average Degree  
$\langle l \rangle$ Average shortest path length  
$\rho$ Pearson Coefficient  
$R\{ k > \bar{k}\}$ Ratio of nodes with larger nodal degree

• Not bad to overlay communications with the lines – relatively short distance
Degree distribution

• Most researchers assume that power grid node degree has a Geometric PDF
  – e.g. [Albert et al. ’04, Rosas-Casals’07]

• Way to highlight:
  **Probability Generating Function (PGF)**
  – For a mixture model

\[ G_X(z) = E(z^X) = \sum_{k} p(X=k) z^k \]

Our analysis result

1. The degree distribution is a mixture of a truncated exponential and finite support random variable
2. The average degree vs. N is O(1)
(a) All buses
(b) Gen buses
(c) Load buses.
(d) Connection buses.
(e) Gen+Load buses.

The zeros are red '++'
Small World conjecture

• Some evidence contradicting it
  – For a SW network with $N$ nodes, to guarantee with high probability a connected network (no isolated component) the scaling laws for the average degree $<k> >> \log N$
  – The average degree in power grids is $\sim$ constant (3-4)

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<td>IEEE-300</td>
</tr>
<tr>
<td>NYISO</td>
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<td>WSCC</td>
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</tbody>
</table>
Scaling Algebraic Connectivity

- Graph Laplacian second smallest eigenvalue
- Values shown in

<table>
<thead>
<tr>
<th>Network</th>
<th>$\lambda_2(L)$</th>
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</thead>
<tbody>
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<td>IEEE-30</td>
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<td>IEEE-57</td>
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<td>WSCC-4941</td>
<td>0.00075921</td>
</tr>
</tbody>
</table>
Plausible topology

- The proposed model that matches this trend is what we call *nested-Small-world* graph
  - IEEE $\rightarrow$ SW subnet 30; NYISO & WECC $\rightarrow$ SW sub-net 300

![Graph diagram showing nested-Small-world topology]

- SW: independent rewiring
- IEEE 300: Correlated rewiring
Impedance distribution

- Absolute values of the impedances
  \[ Z_{pr} = R + jX \approx jX \]
- Prevailingly heavy tailed distributions
- NYISO best fit \( \rightarrow \) clipped Double Pareto Log-normal
Impedance attribution

- Impedance grows with distance: local $\rightarrow$ short; rewires $\rightarrow$ medium; lattice connections $\rightarrow$ long lines

- *RT-nestedSmallWorld*: developed a Matlab code pkg – a helpful tool for many other researchers in the area.
Bus Type Assignment

- Three bus types in a grid:
  - Generation bus
  - Load bus
  - Connection bus
- \textit{RT-nestedSmallWorld}: random assignment according to the bus type ratios

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>Ratio of Bus Types in Real-World Power Networks</th>
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<tbody>
<tr>
<td></td>
<td>( (n, m) )</td>
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<tr>
<td>IEEE-30</td>
<td>(30,41)</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>(57,78)</td>
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<tr>
<td>IEEE-118</td>
<td>(118,179)</td>
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<tr>
<td>IEEE-300</td>
<td>(300, 409)</td>
</tr>
<tr>
<td>NYISO</td>
<td>(2935,6567)</td>
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</tbody>
</table>
Bus Type vs. Node degree

- Correlation between bus types and node degree

**TABLE II**

**Correlation Between Node Degree and Bus Types in Real-World Power Networks**

<table>
<thead>
<tr>
<th></th>
<th>$\langle k \rangle$</th>
<th>$\langle k \rangle_G$</th>
<th>$\langle k \rangle_L$</th>
<th>$\langle k \rangle_G$</th>
<th>$\rho(t, k_t)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>WECC</td>
<td>2.67</td>
<td>-</td>
<td>-</td>
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</table>
### TABLE III

**Bus Types and Clustering Coefficients of Real-World Power Networks and Random Graph Networks**

<table>
<thead>
<tr>
<th>Bus Type</th>
<th>$C(R)$</th>
<th>$C_{all}$</th>
<th>$C_G$</th>
<th>$C_L$</th>
<th>$C_C$</th>
<th>$\rho(t, C_t)$</th>
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<td>0.0801</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The Stochastic Model for Cascading Process

Model the line states as conditionally Markovian given the line flows ``crossing” times

The crossing intervals of line flow $F(t)$ determine the transition rates: $\lambda_0(t)$-random contingency, $\lambda^*(t)$-overload line tripping rate

Stage 0 (intact)

Stage 1 (1 line lost)

Stage 2 (2 lines lost)

The overload threshold

$B'(t)$ incorporates the grid network state
Flow model and level crossing statistics

**Gaussian model for the flows** (tractability)

Gaussian statistics of generation and loads

\[
\mu_P(t) = \begin{bmatrix} \mu_g(t) \\ -\mu_l(t) \end{bmatrix}, \quad C_P(t) = \begin{bmatrix} \Sigma_{gg}(t) & \Sigma_{gl}(t) \\ \Sigma_{lg}(t) & \Sigma_{ll}(t) \end{bmatrix}
\]

We assume loads are independent spatially, non-stationary (cyclostationary)

Temporal correlation of the load is properly captured

The power grid DC flow model

\[
B'(t) = A_t^T \text{diag}\{y_l(t)\} A_t = \tilde{A}_t^T \tilde{A}_t
\]

\[
B'(t) \theta(t) = P(t)
\]

\(A_t\) – the line-node incidence matrix

\(y_l\) – the line admittance

\[
\mu_F(t) = \sqrt{y_l(\tilde{A}_t^T)^\dagger} \mu_F(t)
\]

\[
C_F(t) = \sqrt{y_l(\tilde{A}_t^T)^\dagger} C_F(t) (\tilde{A}_t)^\dagger \sqrt{y_l}
\]

The statistics of line flows

Gaussian process → Rice’s result (1958)
Statistics of level-crossing intervals → line state transition rate \(\lambda_l(t)\)
→ average lifetime
→ probability of overload and distances metrics

CERTS Review, Aug 5-6, 2014
Expected Safety Time of a Line

\[ T_l = \frac{(\bar{\kappa}_l - 1)}{\gamma_l} + E\{\Delta t_l\} \]

- **Expected number of flow crossings after which the line gets tripped**

\[ \bar{\kappa}_l = \frac{[(1 + \beta_l) - (\beta_l - \alpha_l) \rho_l]}{1 - \alpha_l \beta_l} \]

- \( E\{\Delta t_l\} \) is the mean duration of the last interval

\[ E\{\Delta t_l\} = \frac{\Delta T_l^* + \Delta T_l^0}{(1 - \alpha_l \beta_l)} \]

where \( \Delta T_l^* = \frac{(1 - \alpha_l) [\beta_l + (1 - \beta_l) \rho_l]}{\lambda_l^*} \)

and \( \Delta T_l^0 = \frac{(1 - \beta_l) [1 - (1 - \alpha_l) \rho_l]}{\lambda_l^0} \)

- If it is beyond the coherence time of the process, the non stationary metric can be computed using the worst case parameters

\[ T_l^*(t_0) = \frac{(\kappa_l^* - 1)}{\gamma_l^*} + E\{\Delta t_l^*\} \]
IEEE-300 bus system, given the same topology and generation and load statistical settings, the test cases with random bus type assignments tend to have larger expected safety time than that of the realistic grid settings.
Conclusions

• The admittance matrix of power grids has peculiar features that follow clear statistical trends.

• RT-nestedSmallWorld model is designed to capture the accurate statistical properties of real grid topology and electrical parameters.

• It is critical to understand how to design the RT-power grid model better.

• Random assignment of bus types will be replaced with a more accurate one which is consistent with that of realistic grids.
Questions? 😊

Thank You!