Cornell University

PSERC

R & M Project 1A: "Development and Testing of New Tools"

MATPOWER and Tools for Planning and Operation of Power Systems under Uncertainty

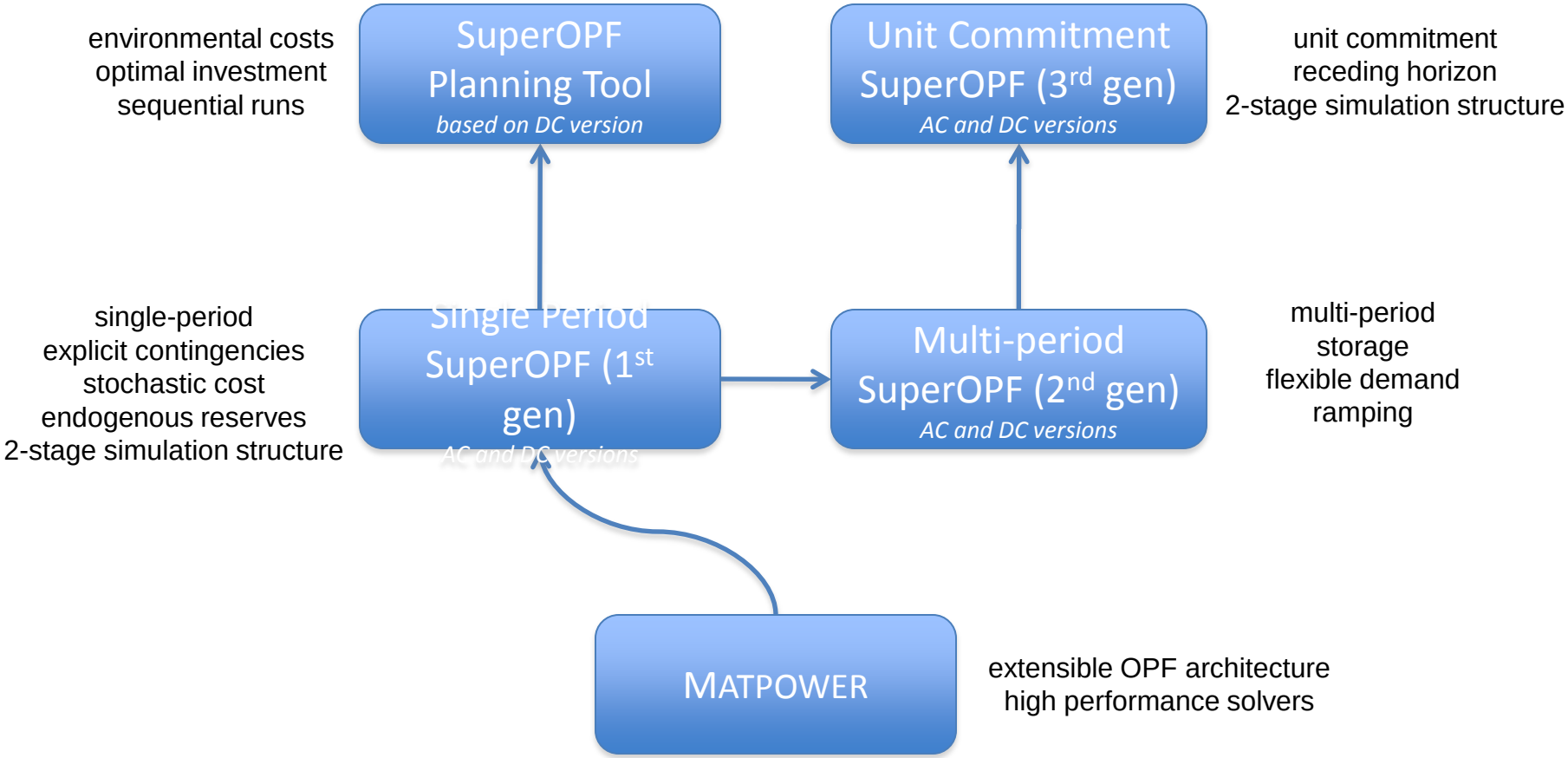
Ray Zimmerman, Carlos Murillo-Sánchez,
Daniel Muñoz-Alvarez, Bob Thomas

CERTS Review, Cornell University
August 5-6, 2014

U.S. DEPARTMENT OF ENERGY

CERTS
CONSORTIUM FOR ELECTRIC RELIABILITY TECHNOLOGY SOLUTIONS

Project Overview



Outline

Summary of Focus of Past Year

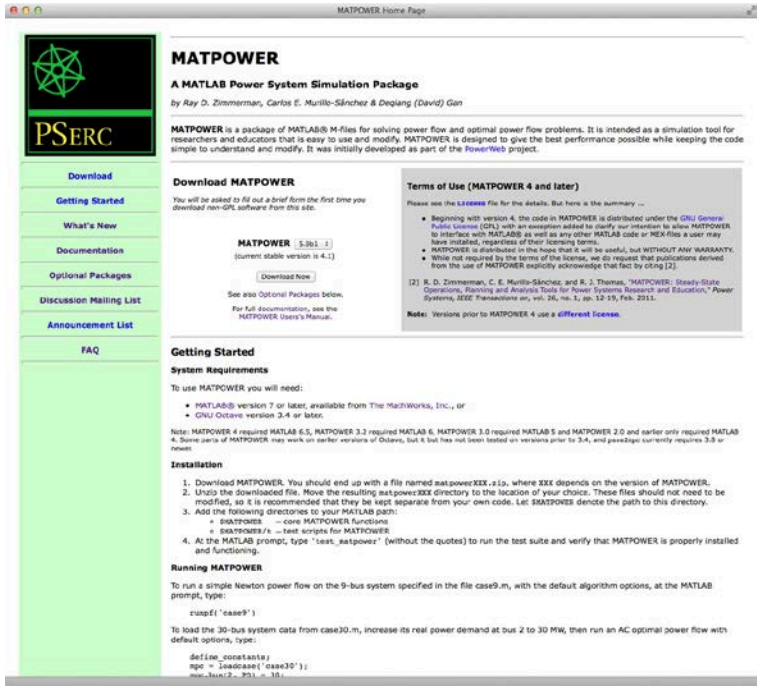
- **MATPOWER** enhancements, new v5 release.
- **Support** for others using SuperOPF tools.
- **Simulation framework** for testing two-stage market structure and receding horizon.
- Multi-period SuperOPF with **Unit Commitment**, completing, refining, testing the 3rd generation code.
- Potential **expansion planning** directions.

MATPOWER

Free, open-source power system simulation environment with extensible OPF and interfaces to state-of-the-art solvers.

http://www.pserc.cornell.edu/matpower/

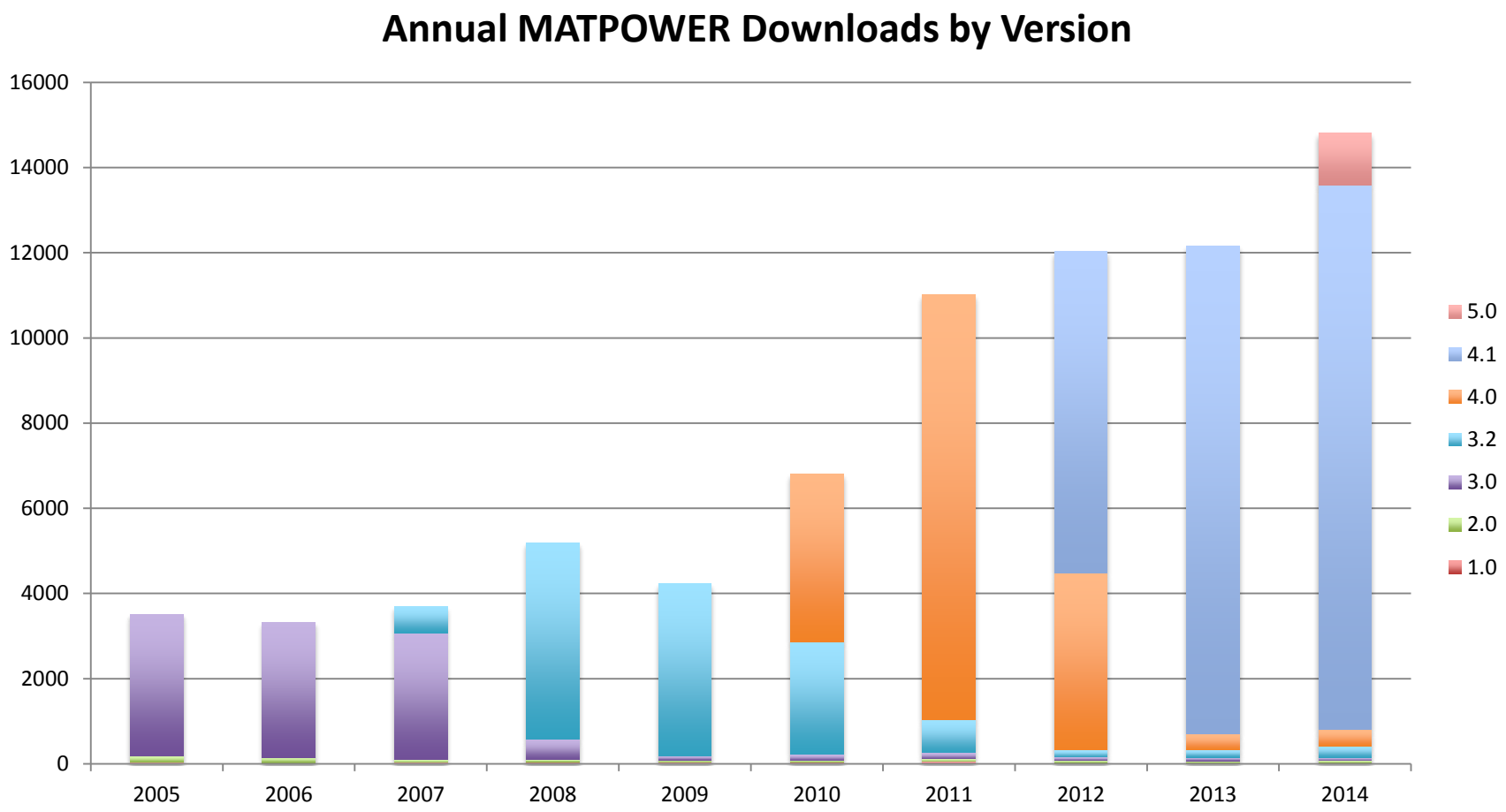
- used worldwide in teaching, research, industry
- momentum continues to grow
- serves as foundation for all tools in this project



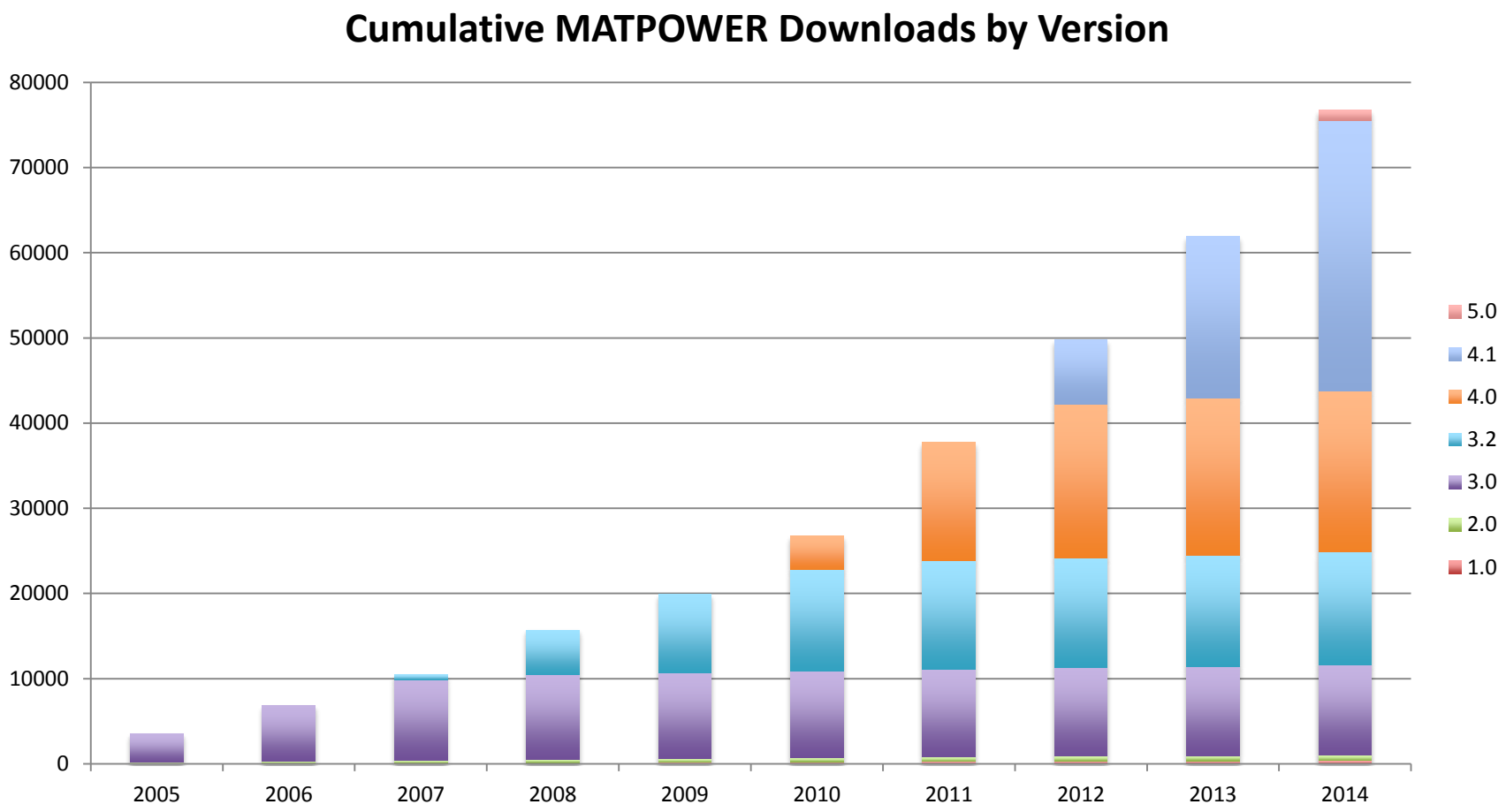
MATPOWER Main Features

- AC & DC power flow
- Continuation power flow
- AC & DC optimal power flow
  - extensible formulation allowing customization via addition of user-defined variables, costs and linear constraints
  - optional co-optimized zonal reserves
  - optional single period heuristic unit (de)commitment
- Unified Matlab API for LP, QP, MILP, MIQP solvers
- GNU Octave compatibility

Annual MATPOWER Downloads



Cumulative MATPOWER Downloads



MATPOWER Papers

- R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER Steady-State Operations, Planning and Analysis Tools for Power Systems Research and Education," *Power Systems, IEEE Transactions on*, vol. 26, no. 1, pp. 12-19, Feb. 2011.
  - **351** citations*
- R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER'S Extensible Optimal Power Flow Architecture," *Power and Energy Society General Meeting, 2009 IEEE*, pp. 1-7, July 26-30, 2009.
  - **51** citations**
- H. Wang, C. E. Murillo-Sánchez, R. D. Zimmerman, and R. J. Thomas, "On Computational Issues of Market-Based Optimal Power Flow," *Power Systems, IEEE Transactions on*, vol. 22, no. 3, pp. 1185-1193, Aug. 2007.
  - **85** citations*

* Scopus 8/2/14

** IEEE Xplore 8/2/14

Highlights of Version 5

MATPOWER 5.0b1 was publicly released July 1, 2014

- redesigned, extensible program **options architecture**
  - lays groundwork for incorporating many additional new features
- import capability for **PSS/E RAW** files *(Yujia Zhu)*
- **continuation power flow** *(Shrirang Abhyankar, Alex Flueck)*
- utility functions to validate and provide **detailed reporting** of case data, including network connectivity
  - extremely useful for debugging connectivity issues with large networks
- utilities for manipulating networks with **islands**
- **applications of SDP** (semi-definite programming) relaxations of power flow equations *(Dan Molzahn)*
  - (globally optimal) solver for SDP relaxation of OPF problem
  - sufficient condition for global optimality of specified OPF solution
  - sufficient conditions for insolvability of power flow equations

Current (MATPOWER 6) Development

- Integration of SuperOPF code, with highest priority on 3rd generation multi-period SuperOPF with unit commitment.
  - assured wide distribution means help for other researchers and increased visibility and opportunities for feedback
  - just beginning this task, **LOTS** of cleanup and documentation work remaining
  - tutorial examples to illustrate flexible implementation:
    - standard deterministic OPF
    - single period secure, stochastic OPF *(1st gen SuperOPF)*
    - multiperiod deterministic OPF (with ramping, storage)
    - multiperiod secure, stochastic OPF *(2nd gen SuperOPF)*
    - deterministic UC w/economic dispatch
    - deterministic UC w/OPF constraints
    - secure, stochastic UC with individual trajectories
    - secure, stochastic UC with full transition probabilities *(full 3rd gen SuperOPF)*

Potential Future Directions

Beyond MATPOWER 6 … *(wish list)*

- Integration of additional SuperOPF capabilities.
- Generalized architecture allowing customization via user-defined *non-linear* constraints and costs.
- Generalized linear PF/OPF *(e.g. linearized around AC operating point, DC OPF with losses)*.
- Additional PF/OPF controls *(e.g. transformer taps, phase shifters)* and modeling *(e.g. FACTS devices, combined cycle plants)*.
- Robustness improvements to built-in interior point solver.
- Full integration of other contributed code:
  - network reduction
  - state estimation
  - transient stability

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MATPOWER & SuperOPF Support

- MATPOWER
  - regular contributions to MATPOWER users e-mail list
  - direct e-mails from users
  - R & M Project 1B: "Essence of Structure Preserving (ESP) Network Reductions for Engineering and Economic Analysis of High Penetration of Renewables" *(Dan Tylavsky)*
    - integration of code to import PSS/E RAW data *(Yujia Zhu)*
    - integration of network reduction code *(Yujia Zhu)*

MATPOWER & SuperOPF Support

- Multi-period SuperOPF
  - R & M Project 2A: "Evaluating the Effects of Managing Controllable Demand & DER" *(Tim Mount)*
    - modeling assistance
    - interpretation of results
    - collaboration on generation of stochastic inputs, including code contributions *(Wooyoung Jeon, Jung Youn Mo, Lindsay Anderson, Amandeep Gupta)*

      (Alberto Lamadrid, Hao Lu)

MATPOWER & SuperOPF Support

- SuperOPF Planning Tool
  - R & M Project 2E: “Mapping Energy Futures: SuperOPF Planning Tool” *(Bill Schulze)*
    - implementation of dispatchable HVDC lines
    - incorporating model of HVDC line from HQ to NYC
    - refining of network reductions
    - troubleshooting and debugging new models
      - generator data vs. network
      - interpretation of results

      (Biao Mao, Dan Shawhan, Yujia Zhu others)

Publications

- Carlos E. Murillo-Sánchez, Ray D. Zimmerman, C. Lindsay Anderson and Robert J. Thomas, **"A Stochastic, Contingency-Based Security-Constrained Optimal Power Flow for the Procurement of Energy and Distributed Reserve"**, *Decision Support Systems*, vol. 56, pp. 1-10, Dec 2013, ISSN 0167-9236.
  - Available: http://dx.doi.org/10.1016/j.dss.2013.04.006

- Carlos E. Murillo-Sánchez, Ray D. Zimmerman, C. Lindsay Anderson and Robert J. Thomas, **"Secure Planning and Operations of Systems with Stochastic Sources, Energy Storage and Active Demand"**, *Smart Grid, IEEE Transactions on*, vol. 4, no. 4, pp. 2220-2229, Dec. 2013.
  - Available: http://dx.doi.org/10.1109/TSG.2013.2281001

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Critical Aspects of New Tool Development & Testing

Simulation Environment

defines time structure, data/information flow patterns between …

- *multiple decision stages (e.g. day-ahead UC, 5-min dispatch/pricing)*
- *sequential solves of a given stage*
- *actual operation*

modeling

devices, networks, storage, uncertainty, markets

data

solvers

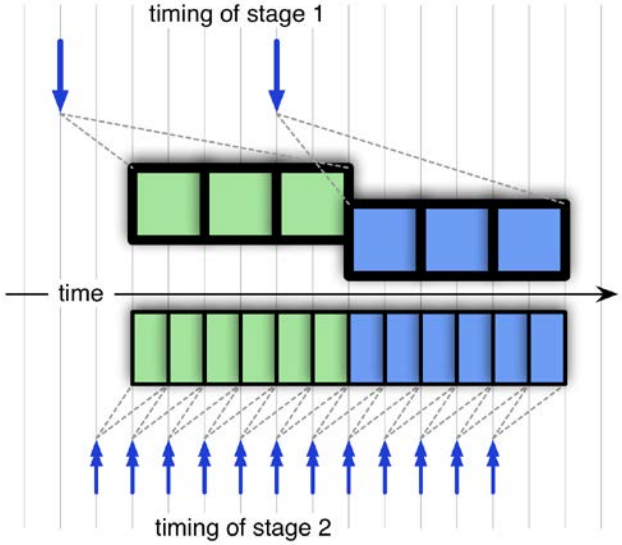
problem formulation

algorithms

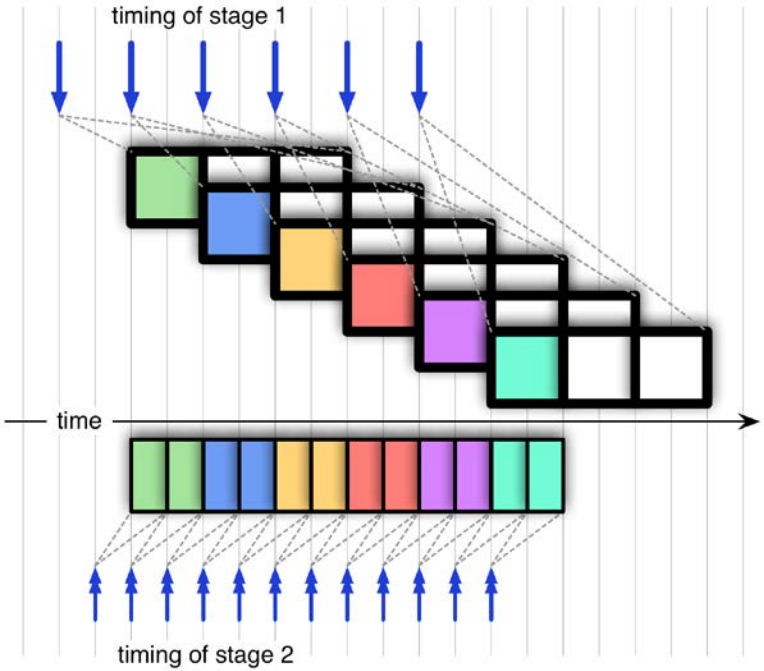
Simulation Environment

- Completed some planning, design work in this direction.
- Focus on other aspects has delayed building out this aspect of the SuperOPF framework.
- So far users have been creating their own ad hoc environments.
- Now working on providing unified framework for two-stage market benchmarking, receding horizon simulations and more.

Day-at-a-Time vs. Receding Horizon



Traditional Approach – Stage 1 runs *once-per-day*, finds hourly solution for *full day*; stage 2 runs intra-hour, finds single period solution subject to *day-ahead* contracts.



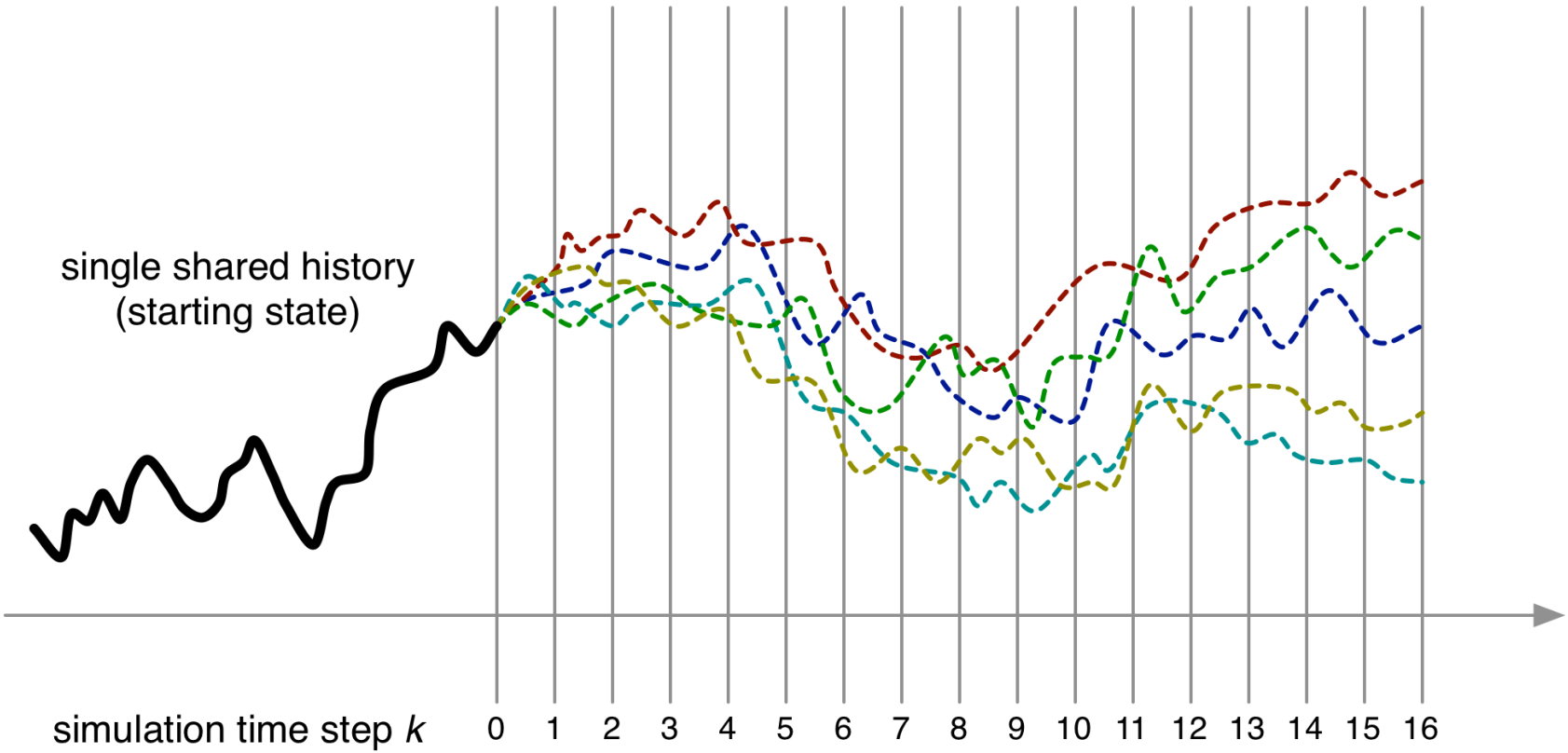
Receding Horizon Approach – Stage 1 runs *hourly*, finds solution for *first hour* with hourly full-day look-ahead; stage 2 runs intra-hour, finds single period solution subject to *hour-ahead* contracts.

Unified Framework

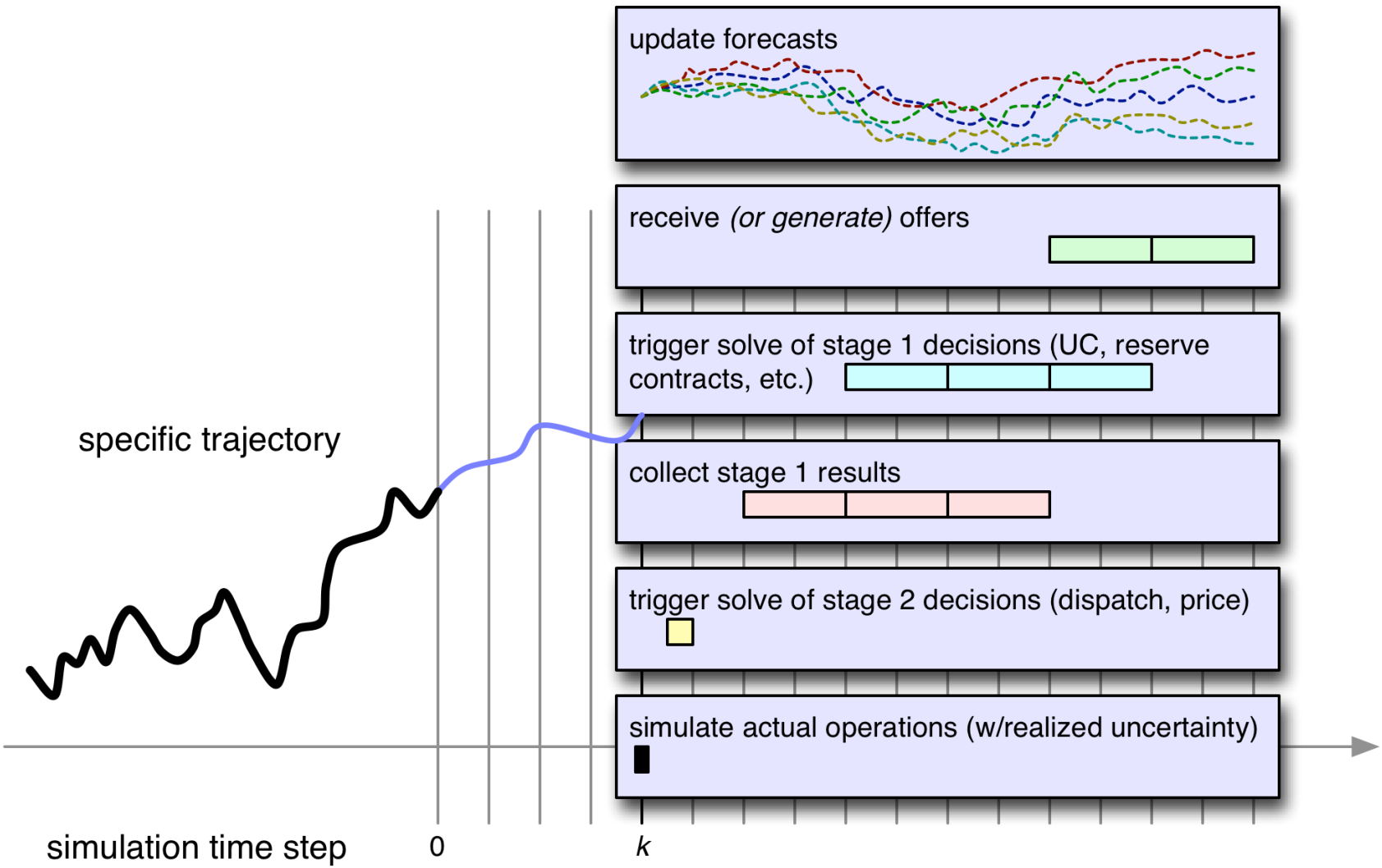
- Goal of a general structure, facilitating:
  - two stage benchmarking
  - receding horizon testing
  - simulation stage (representing what actually happens in the real world) separate from 2nd stage decisions
  - explicit structure of uncertainty revelation.
  - other tasks, such as:
    - forecasting
    - offer generation

Simulation of Multiple Trajectories

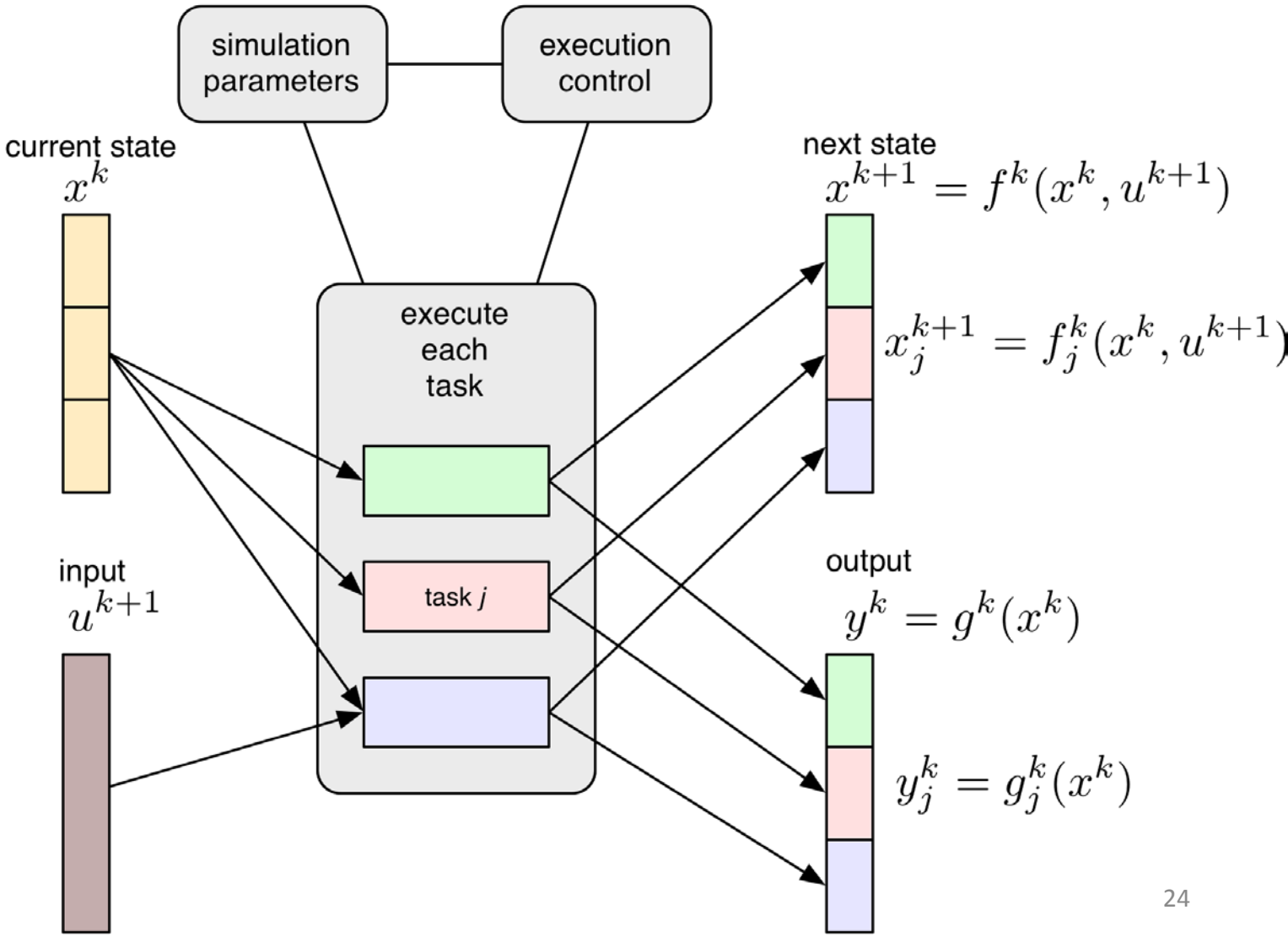
(potentially in parallel)



Step *k* of Given Trajectory

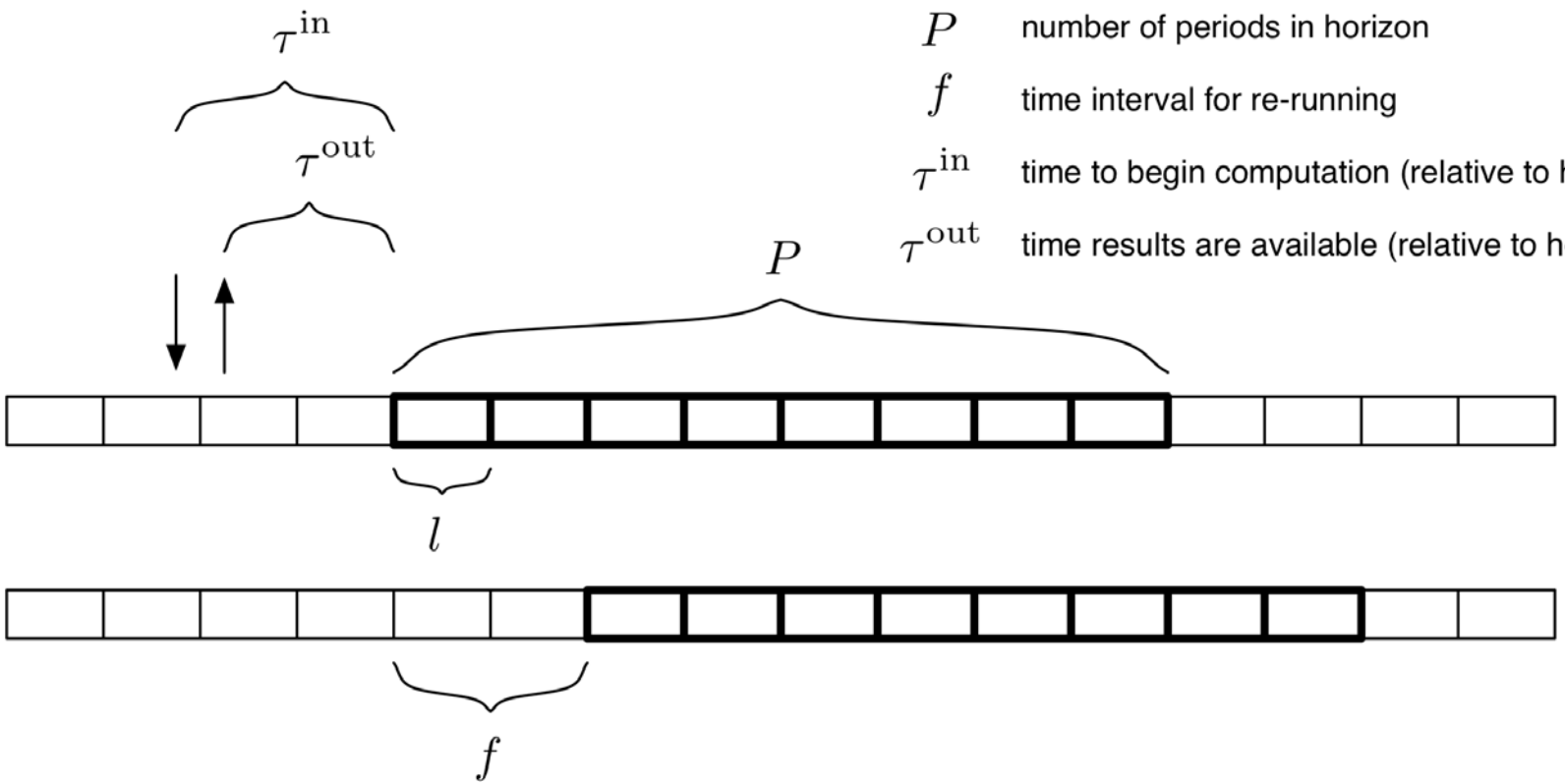


State Update Diagram



Task Specific Parameters

l	length of each period
P	number of periods in horizon
f	time interval for re-running
τ^{in}	time to begin computation (relative to horizon)
τ^{out}	time results are available (relative to horizon)



Stage 1 to Stage 2 Data Flow

Stage 1

- multi-period look-ahead
- e.g. hourly resolution
- solving for:
  - unit commitments
  - reference dispatch
  - reserve capacity contracts
  - ramp capacity contracts
  - storage schedule

Stage 2

- single period *(or shorter look-ahead)*
- e.g. 5 min resolution
- fixed:
  - unit commitments
  - reference dispatch
  - generator ranges
  - ramping limits
  - storage dispatch limits, value of leftover storage

Goals

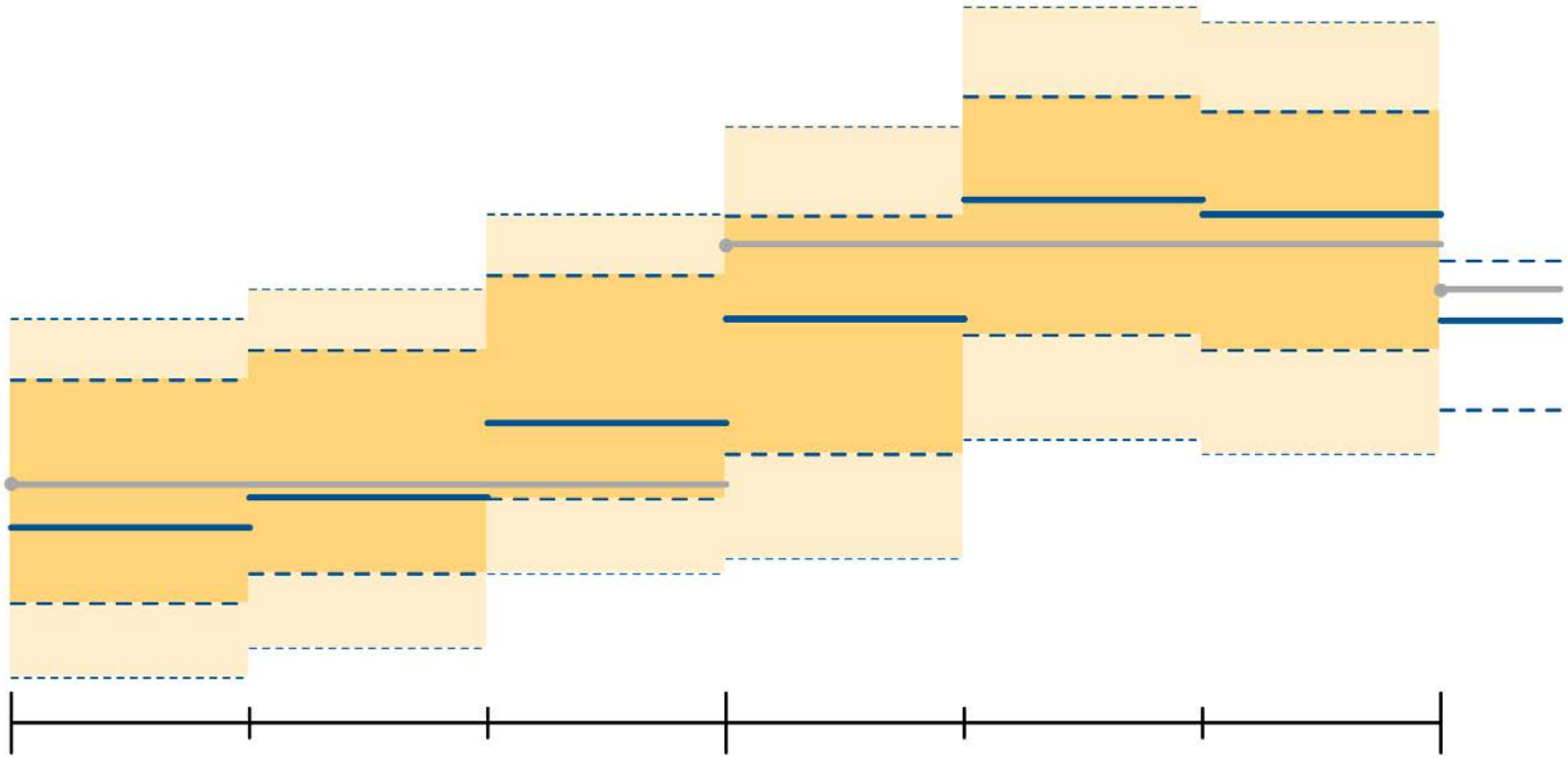
- maximize flexibility to take advantage of new info

  current state, improved forecasts, etc.
- respect look-ahead plan

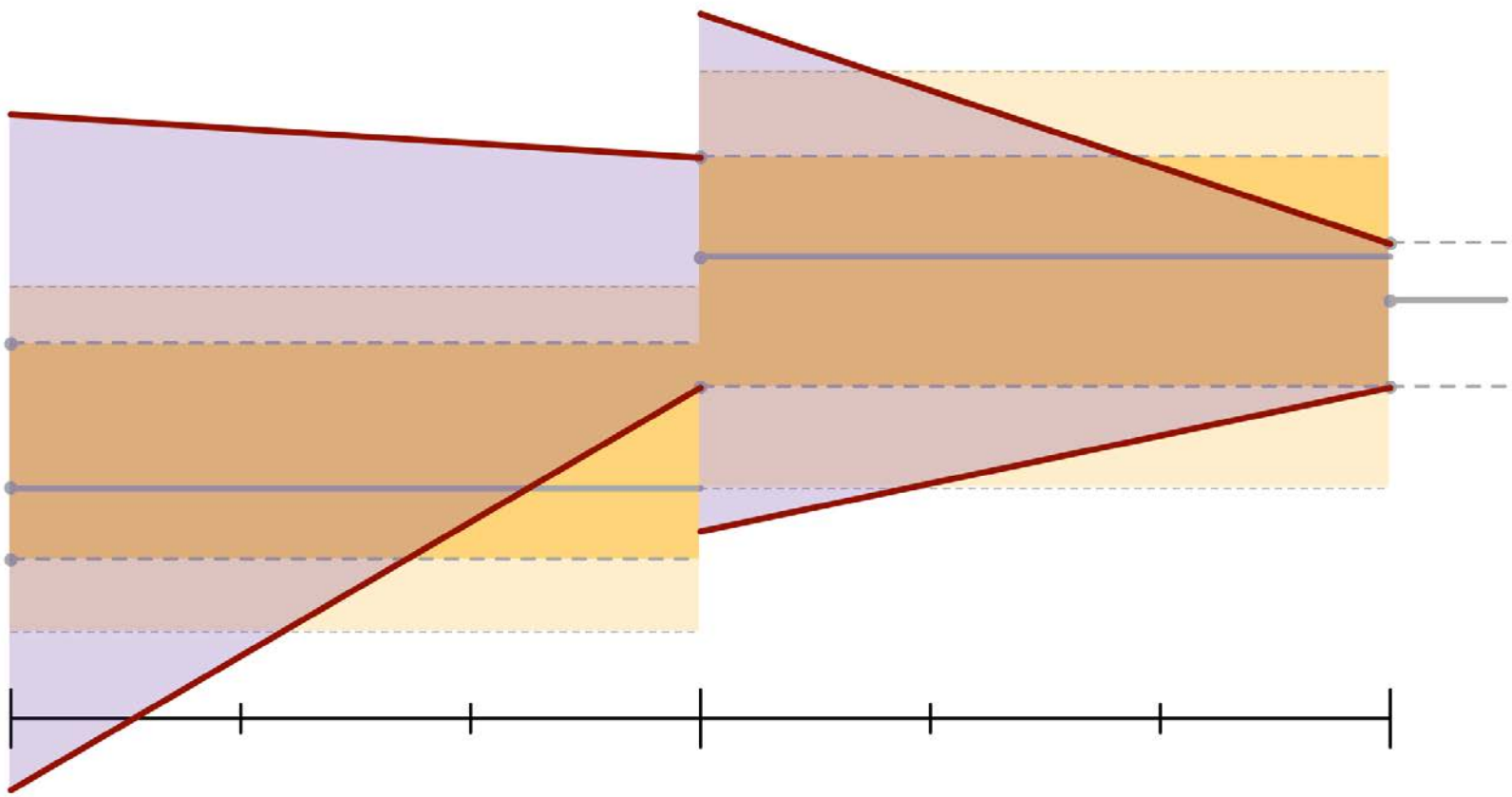
  stage 1 solution is our only link to the info (ramping, storage) obtained by looking-ahead in stage 1

Reference Dispatch and Reserves

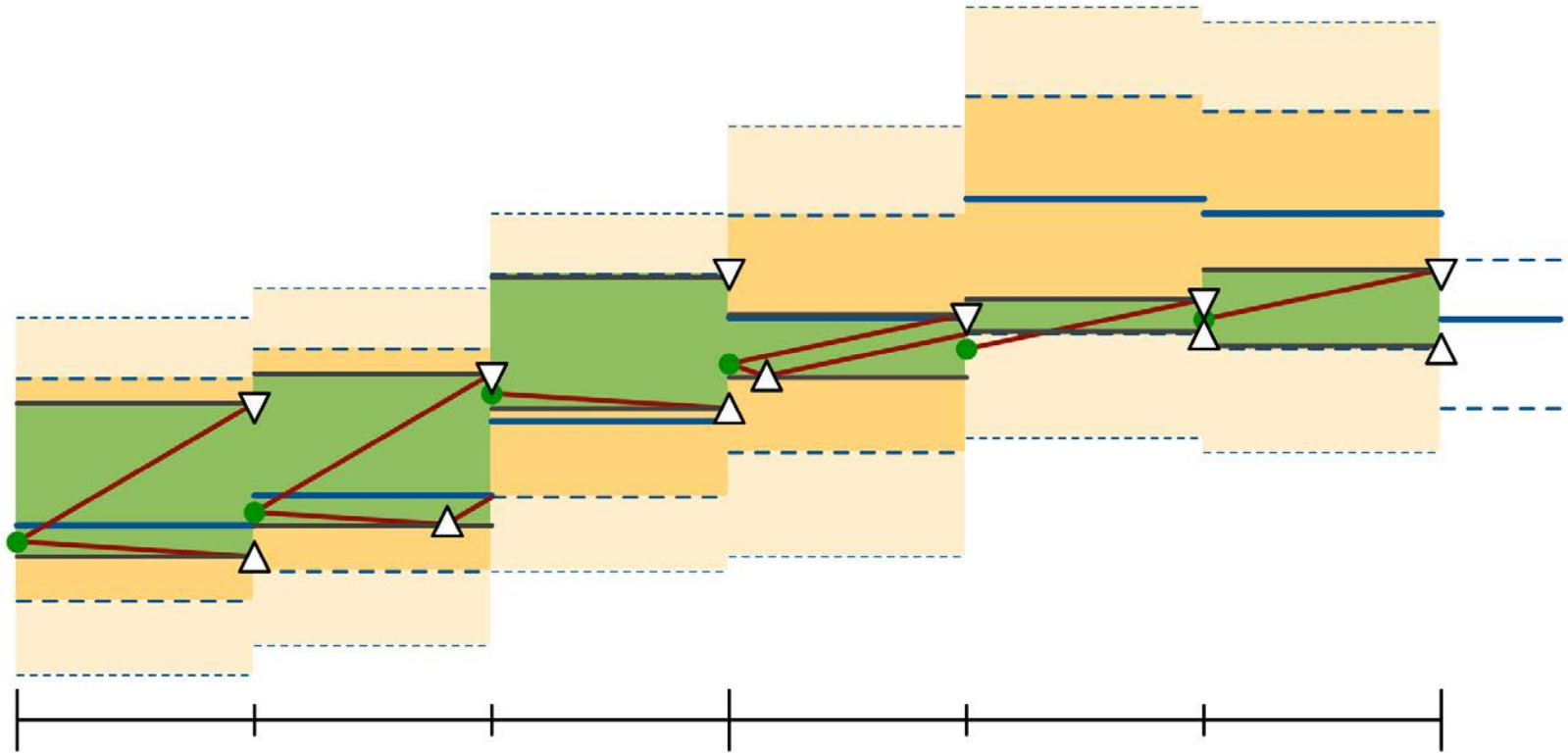
Dispatch Ranges for Intact & Contingency States



Ramp Reserve Contracts



Second Stage Dispatch Restrictions



Other Design Aspects

- similar details involved in converting stage 1 storage plan into appropriate single period restrictions on storage dispatch
  - maximize flexibility to use new information
  - respect stage 1 look-ahead information
- input/output data and file structures
  - forecasts
  - offers
  - optimization solutions
- parallelization

Summary of FY14 Focus

- Incorporate 3rd gen multi-period SuperOPF with **unit commitment** into **MATPOWER**.
- Create **tutorial examples** of various applications.
- Finish design/implementation of **unified simulation framework**.
- **Benchmark** multi-period SuperOPF vs. deterministic multi-period model using **two-stage market** structure under wind uncertainty.
- Compare **receding horizon** approach to traditional "day-at-a-time".

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Unit Commitment

- injection limits and commitments

$$u^{ti} P_{\min}^{tijk} \leq p^{tijk} \leq u^{ti} P_{\max}^{tijk}$$
$$u^{ti} Q_{\min}^{tijk} \leq q^{tijk} \leq u^{ti} Q_{\max}^{tijk}$$

- startup and shutdown events

$$u^{ti} - u^{(t-1)i} = v^{ti} - w^{ti}$$

- minimum up and down times

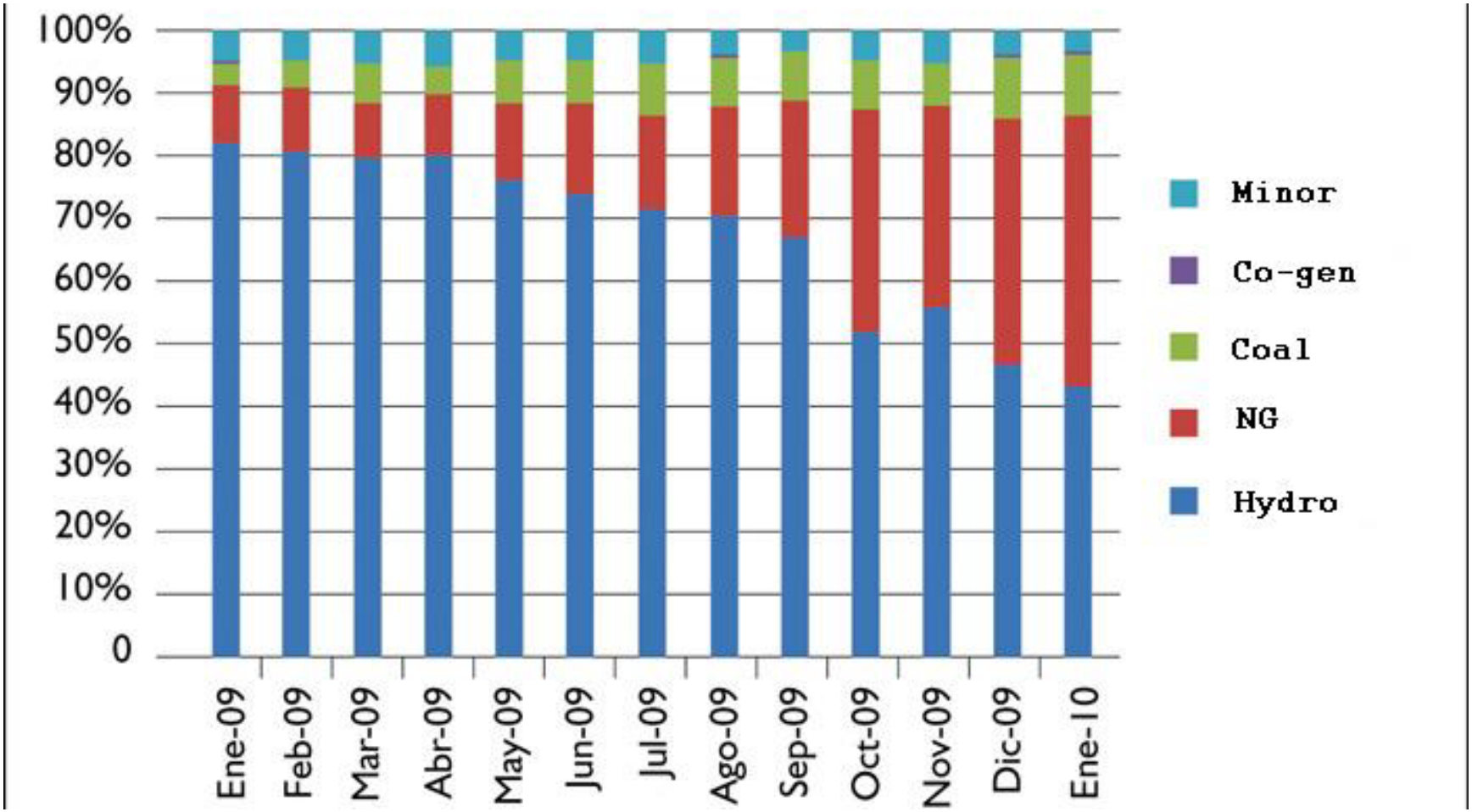
$$\sum_{y=t-\tau_i^+}^{t} v^{yi} \leq u^{ti}, \qquad \sum_{y=t-\tau_i^-}^{t} w^{yi} \leq 1 - u^{ti}$$

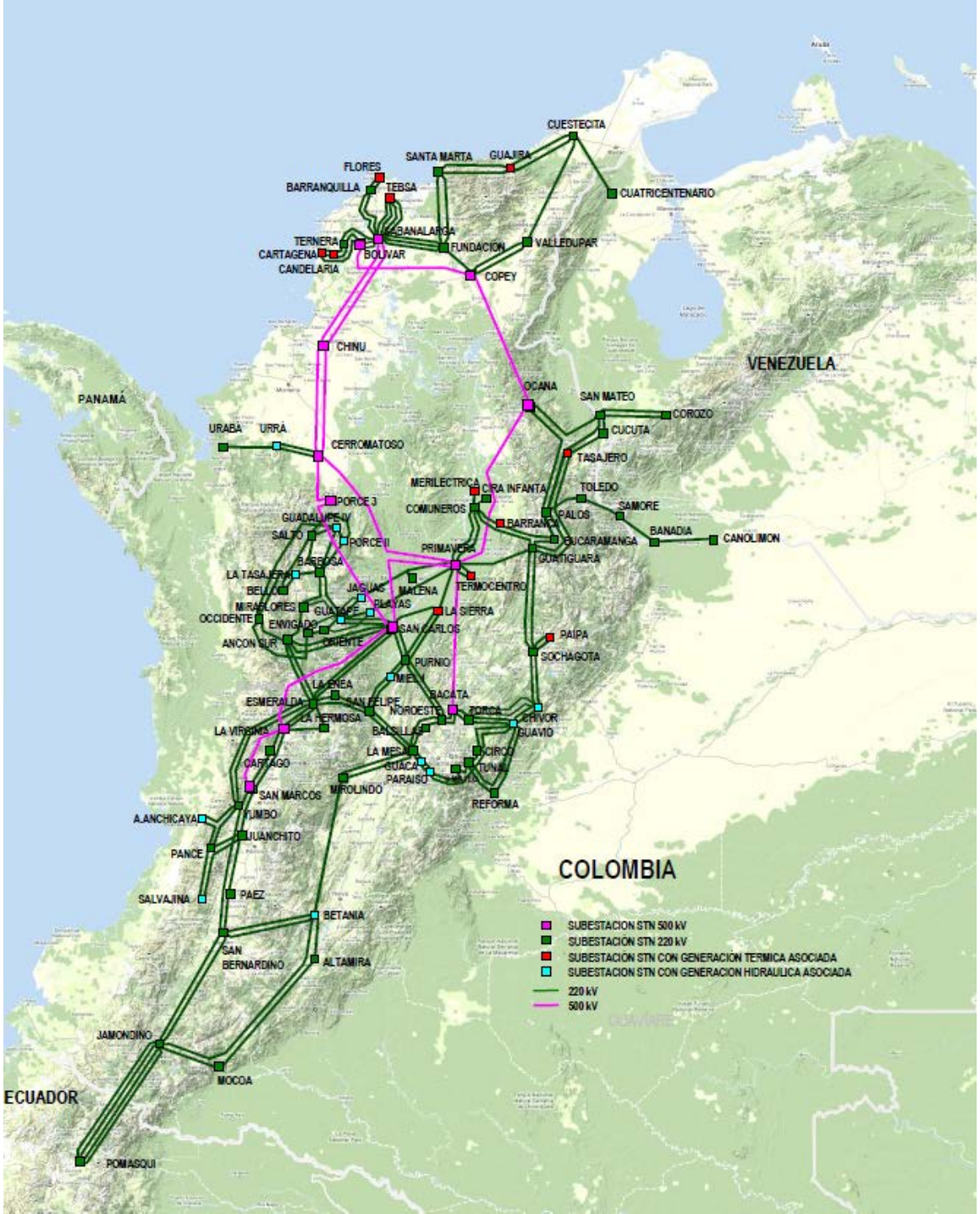
- In practice, only u variables need to be defined as binary.
- An u variable shuts down all injections related to a given generator in a given time slice (all scenarios, all contingencies and base cases).
- On input, a unit can have “available for commitment decision”, “forced on” or “forced off” status.

Colombian system study

- Purpose: to exert all of the capabilities of the software on a real system. Wind uncertainty, storage model for hydro, ESS, unit commitment.
- Real system data plus two hypothetical features:
  - Wind farm in the site with the best characteristics; Colombia's only experimental farm is there.
  - Thermal storage in the Caribbean region (air conditioning needs there are projected to increase substantially due to climate change)
- Depending on the season and whether El Niño is ongoing, 50 to 80% of the energy served is hydro in Colombia.

Generation mix 2009

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SANTA MARTA
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ENVIGADO
SAN CARLOS
PAIPA
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SOCHAGOTA
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LA PREA
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JUANCHITO
PANCE
COLOMBIA
SALVAJINA
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BETANIA
SAN BERNARDINO
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SUBESTACION STN CON GENERACION HIDRAULICA ASOCIADA
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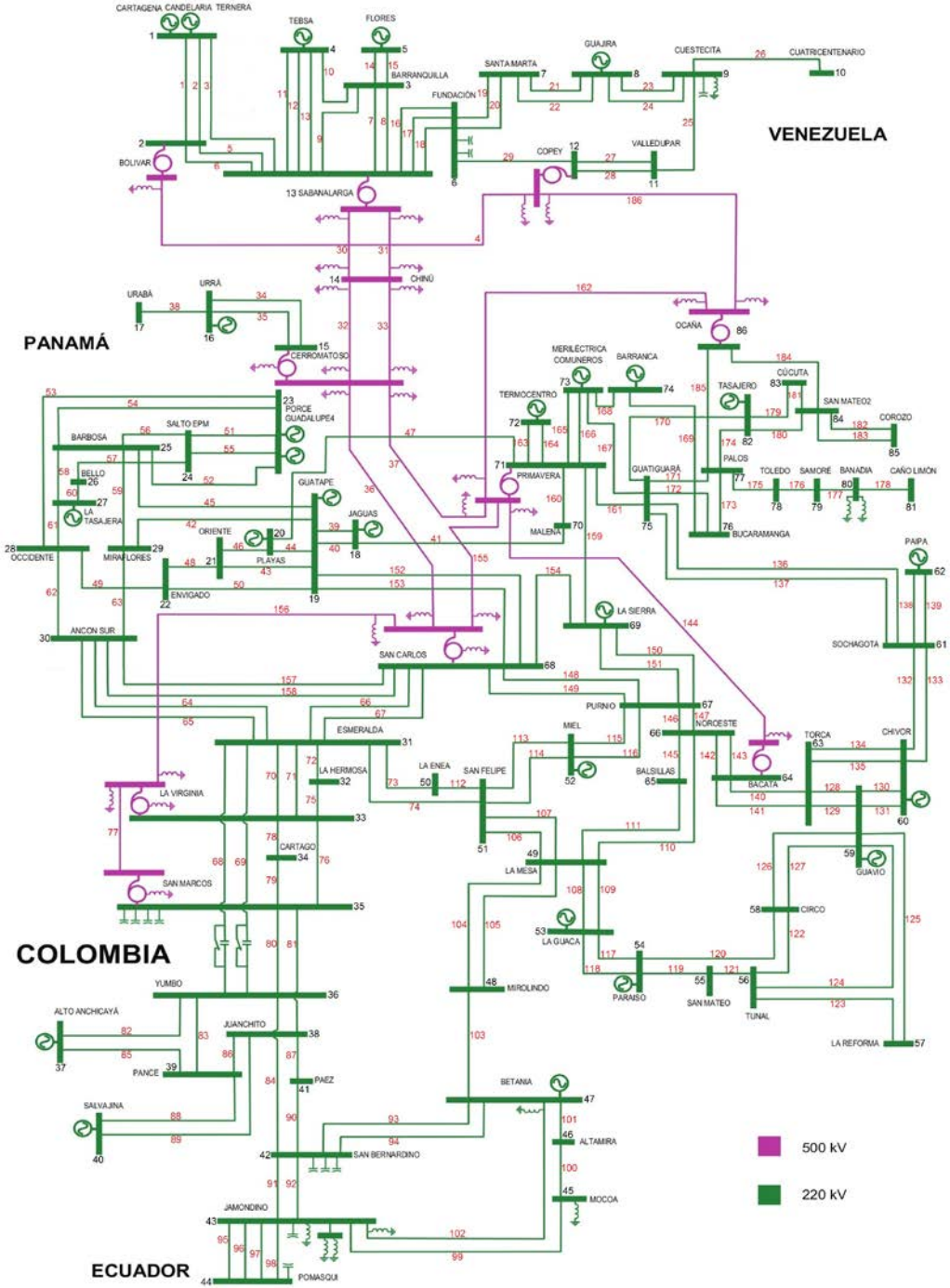
POMASQUI

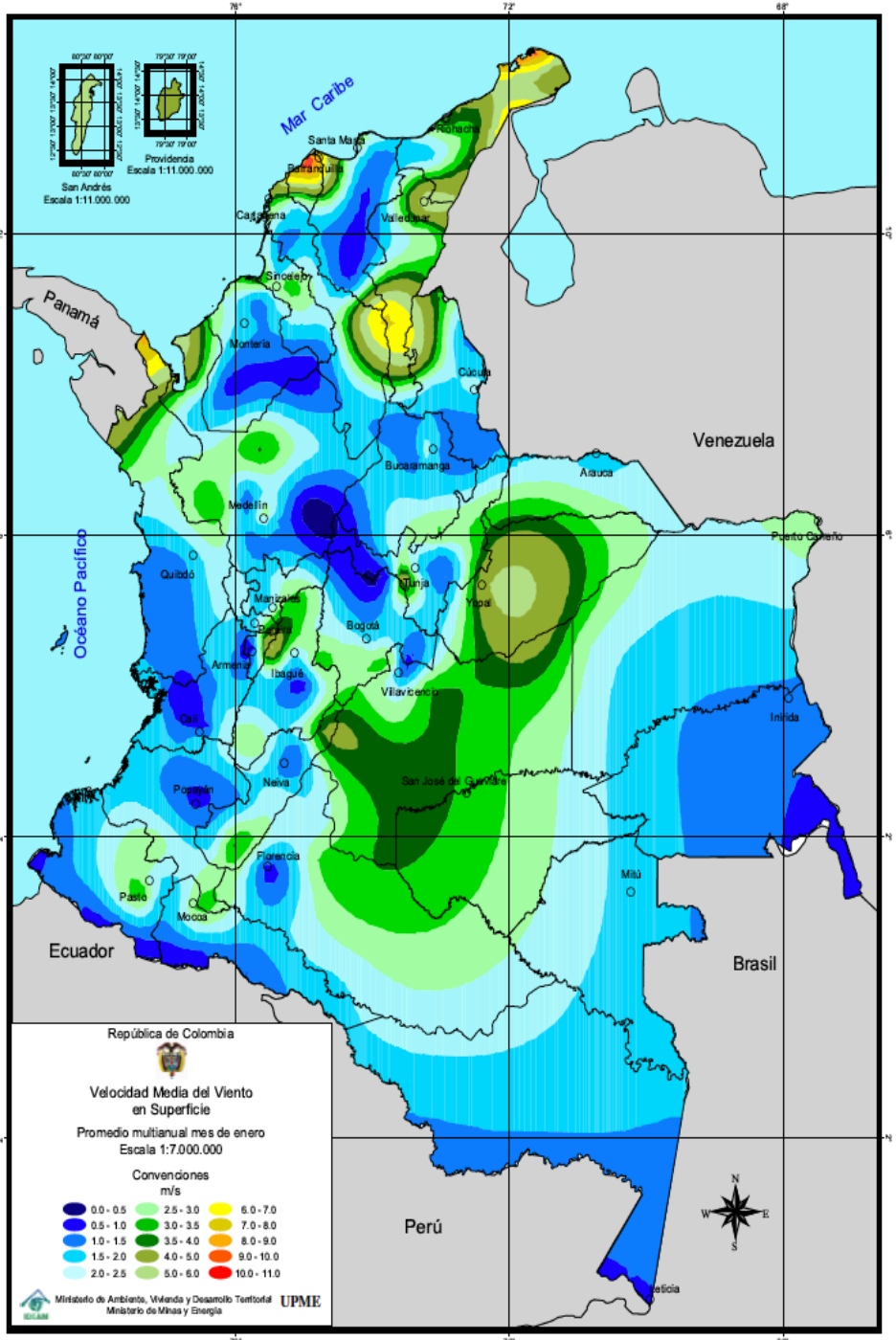
DIAGRAMA UNIFILAR SISTEMA DE TRANSMISIÓN ACTUAL



Technical conditions of study

- Hydro constrained to provide 50% of served energy (dry season; also best wind season).
- 50% of load in the Caribbean deemed to be time-flexible (to simulate thermal storage).
- $Wind < $Hydro < $Coal < $NGCC < $NG < $Oil
- Wind installed capacity is varied from 200MW to 1000MW.
- 4 wind scenarios, 10 worst contingencies.
- Wind statistics: not known completely, but we have seasonal/hourly averages, standard deviations and skews from an existing study.

- Heat rates for the major thermal plants are known, as well as average offer price by generation technology.
- Thermal plants offer at significantly higher prices than their heat rate and fuel cost would allow to guess (offers public after 1 month).
- Unit commitment characteristics inferred using the same methodology as in FERC's unit commitment test cases.
- 177 individual units in 95 plants; 86 buses, 186 branches in transmission network with 110KV – 500KV lines.


Mar Caribe
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Arauca
Medellín
Puerto Carreño
Océano Pacífico
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Ministerio de Minas y Energía
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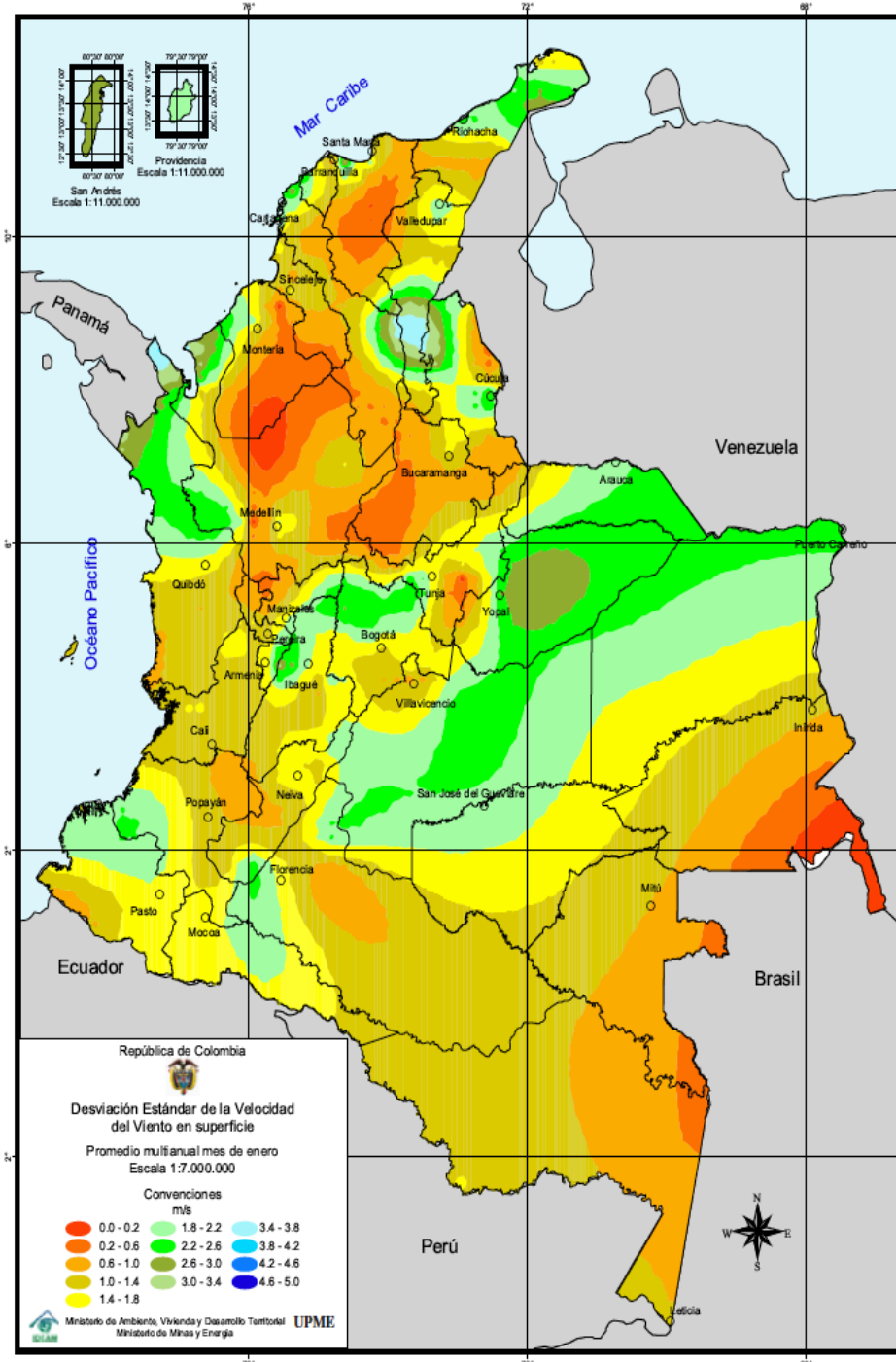
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Medellín
Puerto Carreño
Océano Pacífico
Quibdó
Manizales
Pereira
Tunja
Yopal
Bogotá
Armenia
Ibagué
Villavicencio
Cali
Inírida
Popayán
Neiva
San José del Guaviare
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Perú
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San Andrés
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Escala 1:11.000.000
República de Colombia
Desviación Estándar de la Velocidad
del Viento en superficie
Promedio multianual mes de enero
Escala 1:7.000.000
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3.0 - 3.4
3.4 - 3.8
3.8 - 4.2
4.2 - 4.6
4.6 - 5.0
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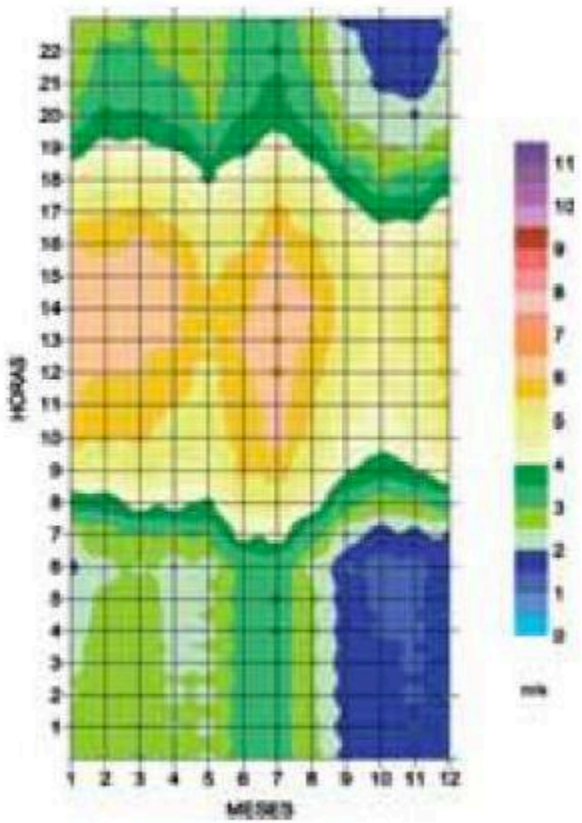
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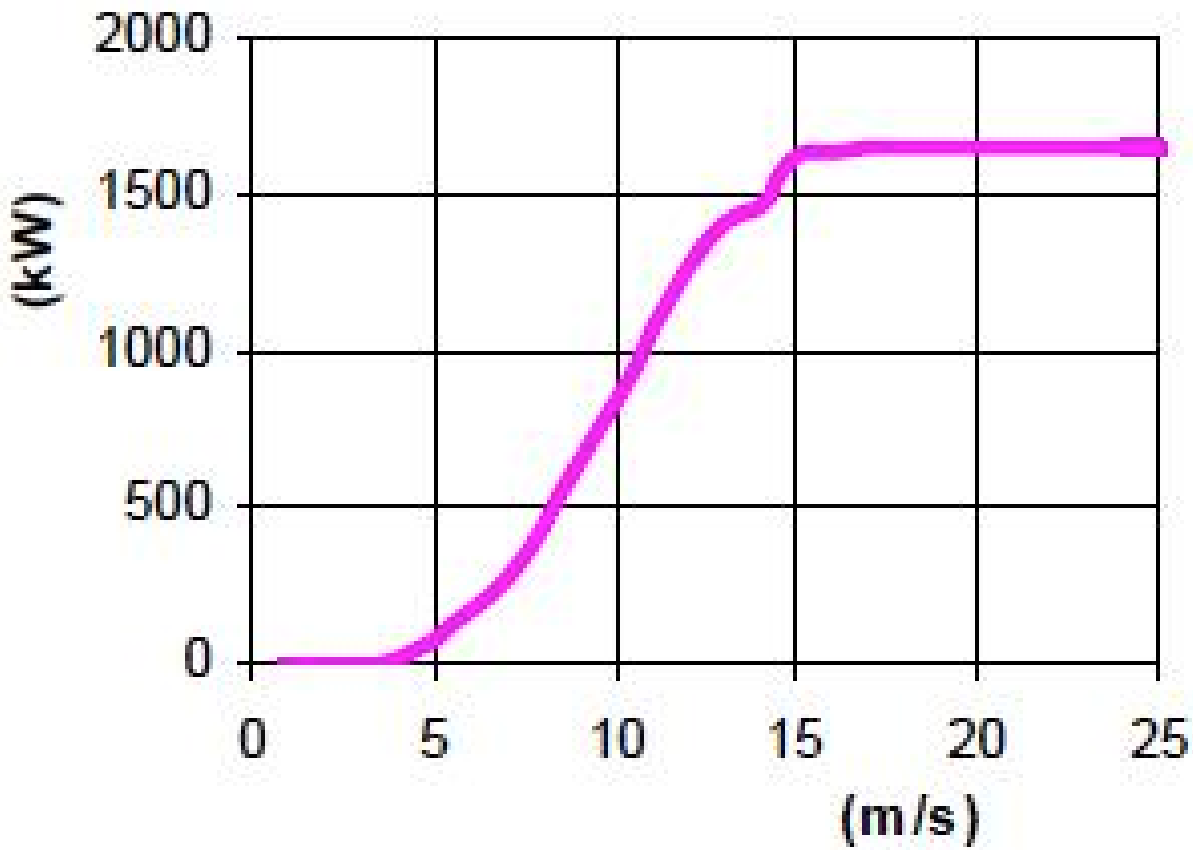
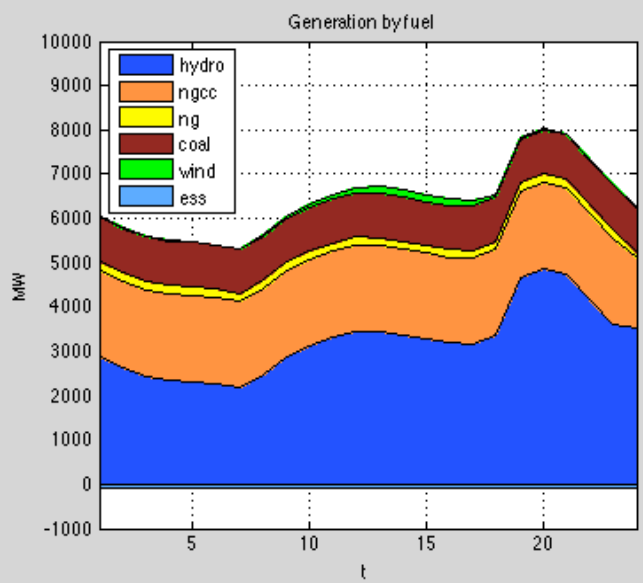
Power curve



Colombia's sole wind farm

- Experimental.
- Pmax = 19.5MW < 20MW.
- Planned capacity factor: 42%.
- Actual obtained: 36%.
- Wind potential in Guajira region: > 10 Gw.
- Wind seasonal availability mostly complementary to hydro; wind higher in dry season.

200 MW wind


Generation by fuel
hydro
ngcc
ng
coal
wind
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MW
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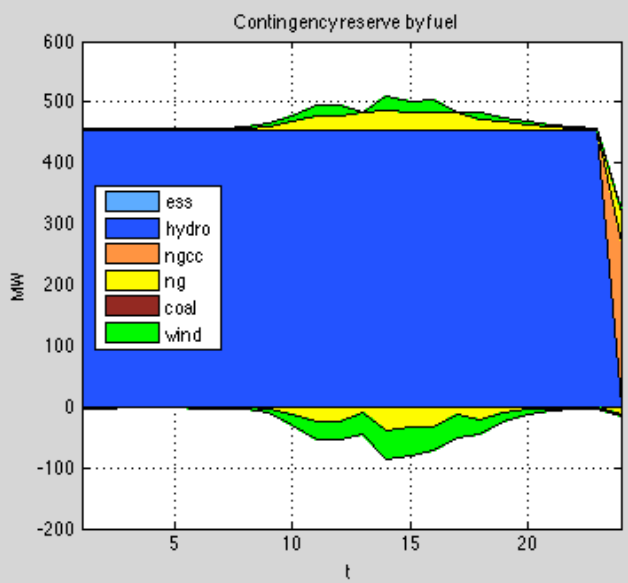
Contingency reserve by fuel
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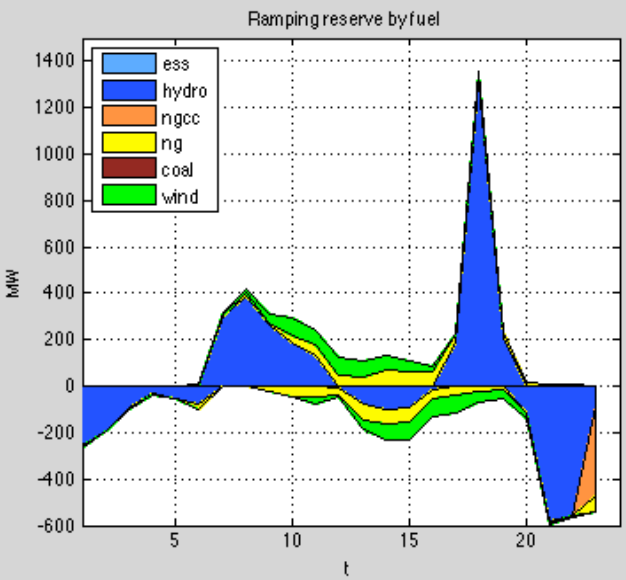
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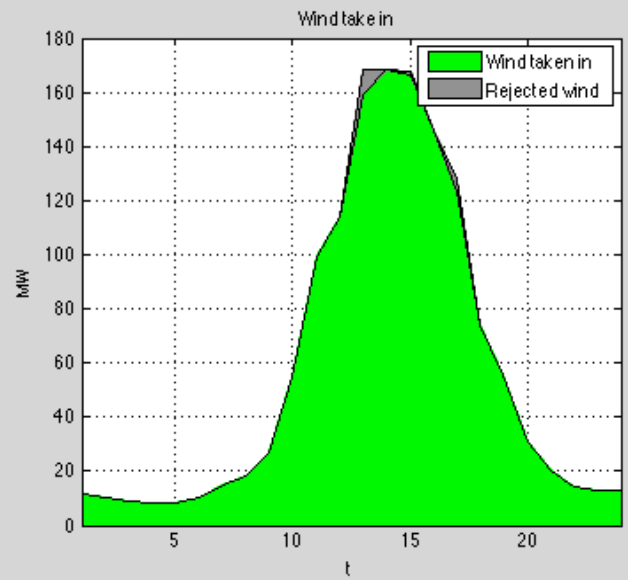
Wind take in
Wind taken in
Rejected wind
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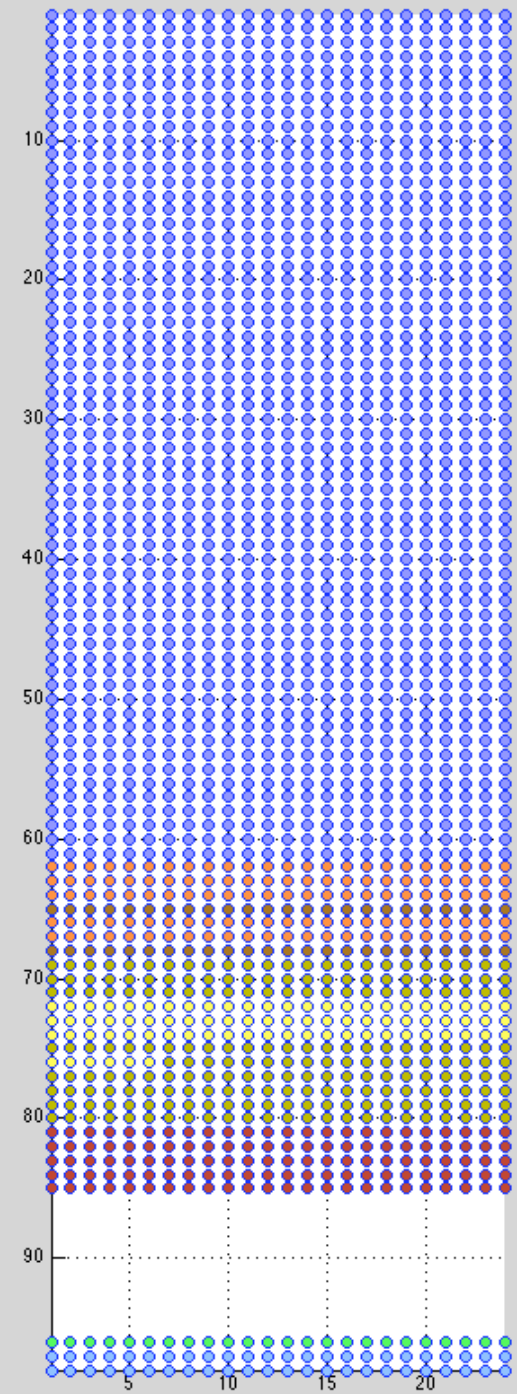
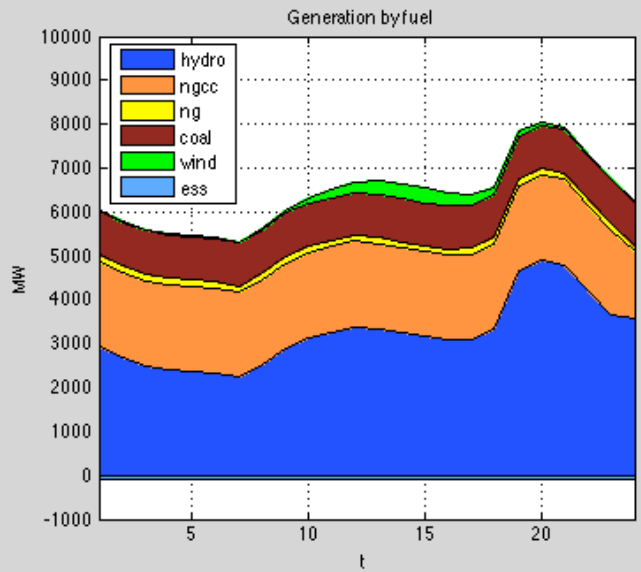


400 MW wind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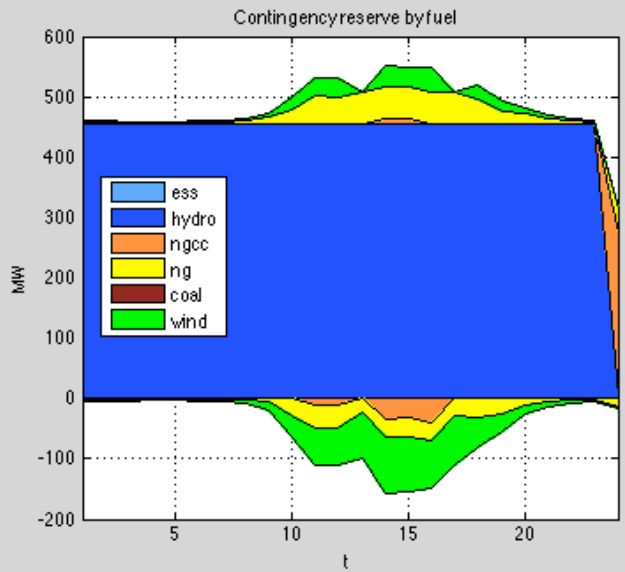
Contingency reserve by fuel
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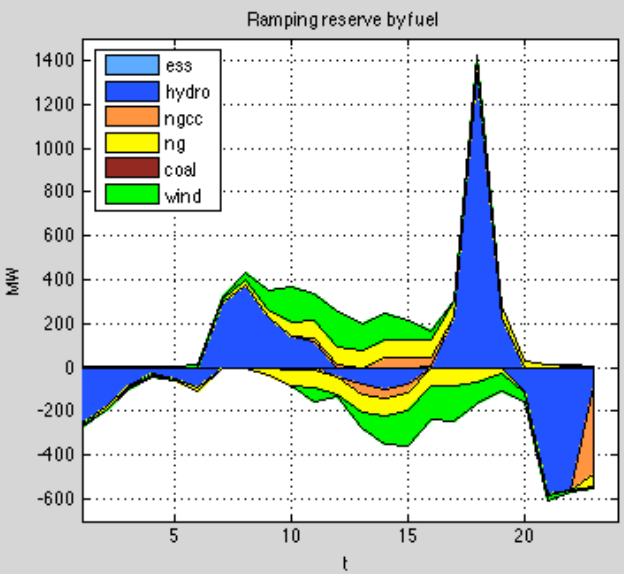
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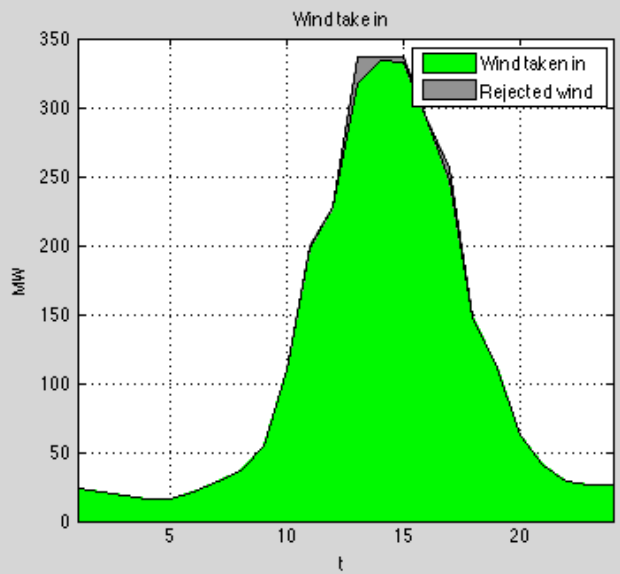
Wind take in
Wind taken in
Rejected wind
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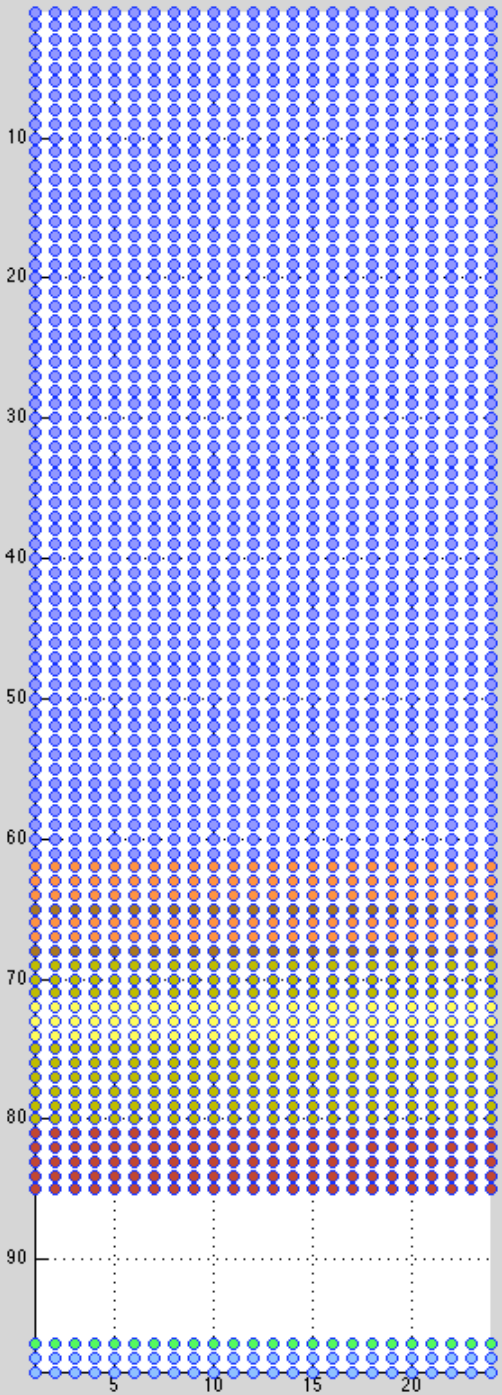
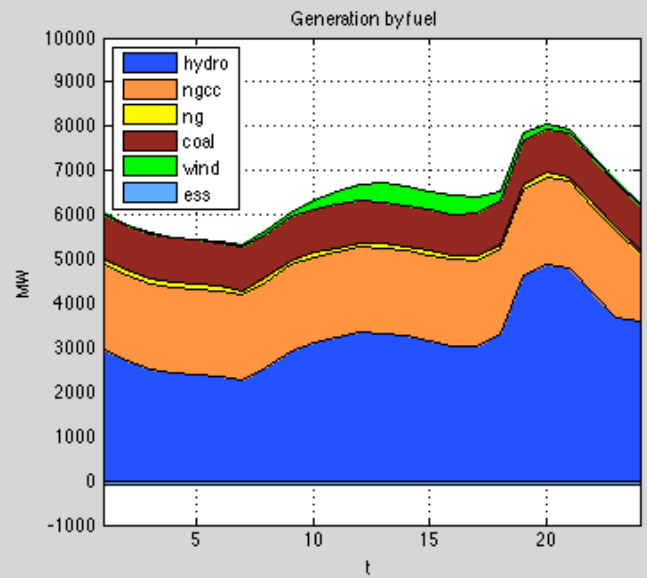


600 MW wind


Generation by fuel
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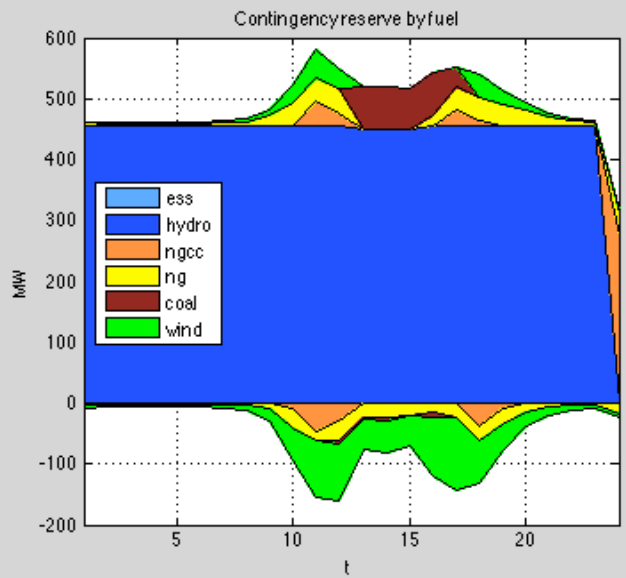
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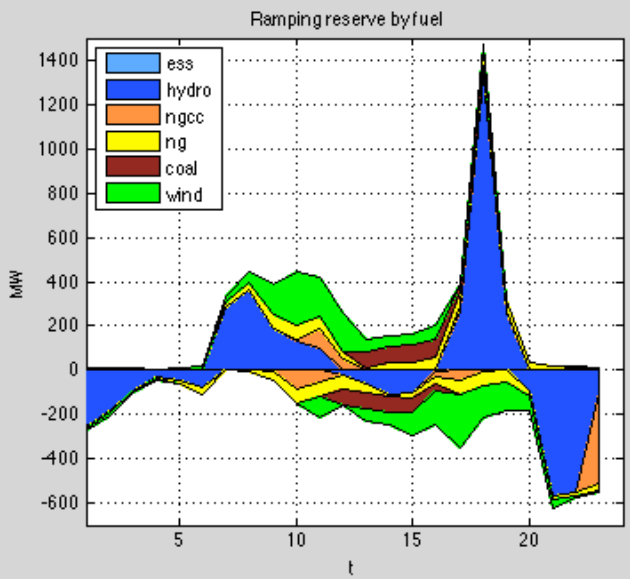
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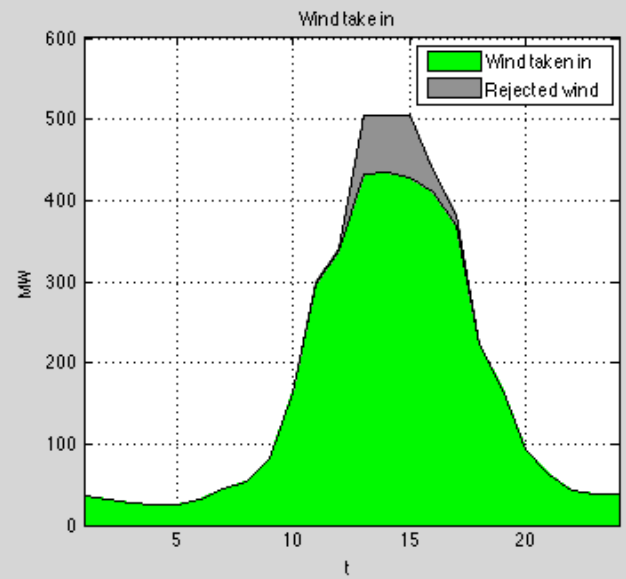
Wind take in
Wind taken in
Rejected wind
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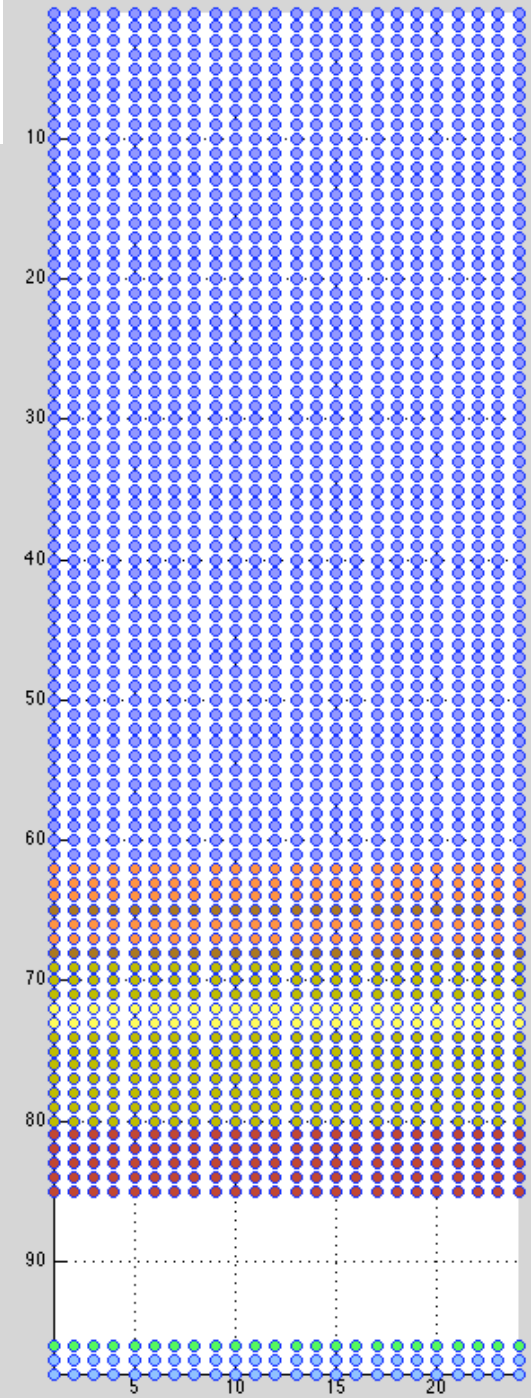
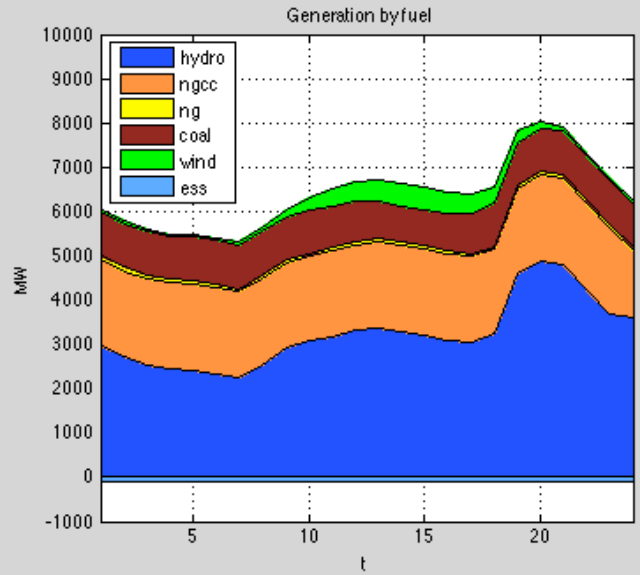


1000 MW wind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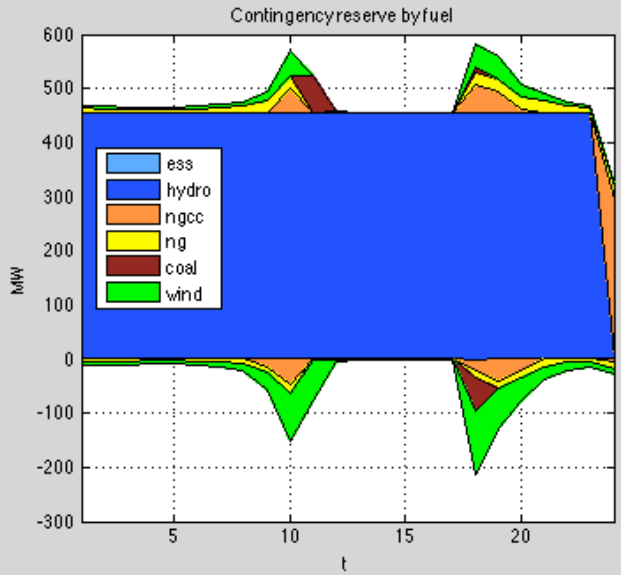
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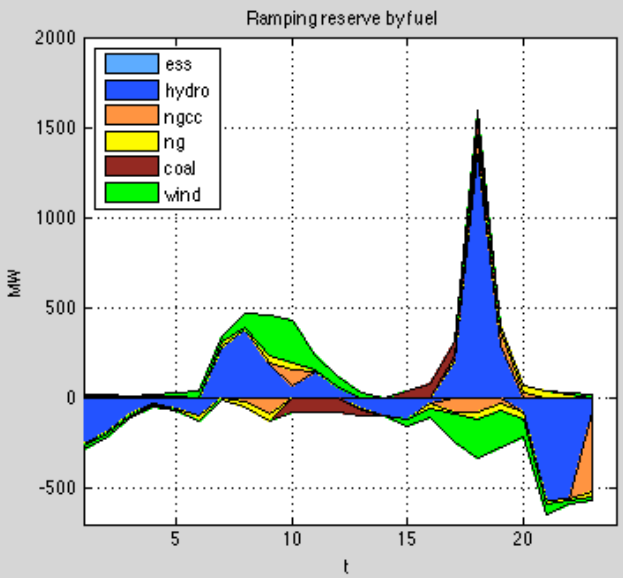
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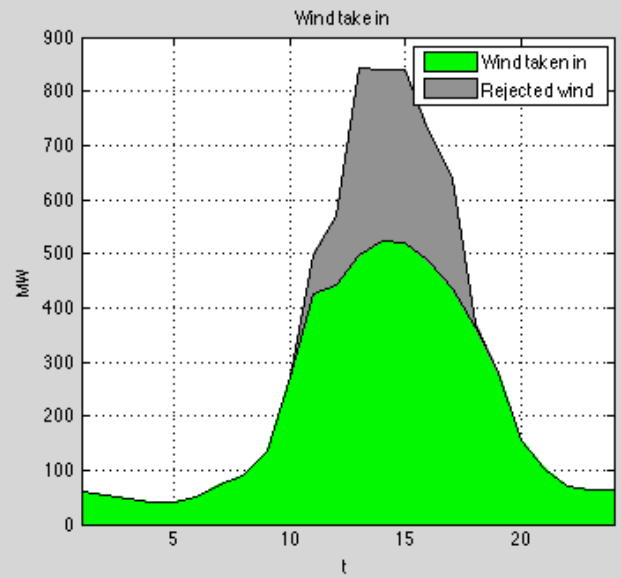
Wind take in
Wind taken in
Rejected wind
MW
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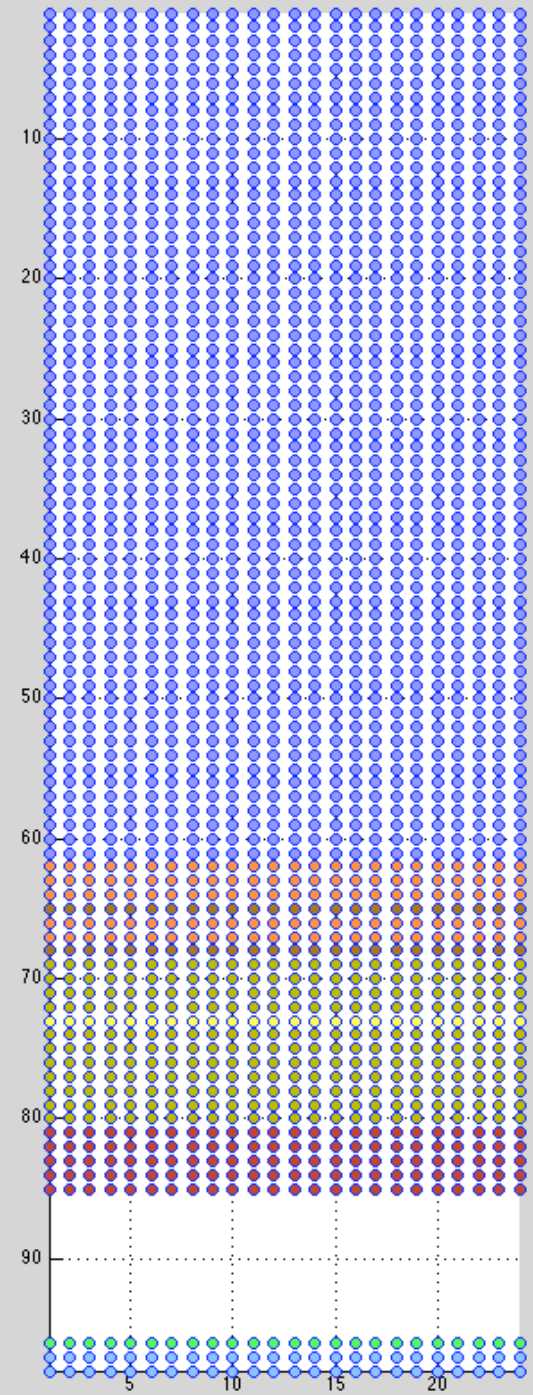


Study conclusions:

- Maximum expected wind take-in is about 500MW due to security concerns (coincides with the figure provided by the Colombian Ministry of Energy).
- Transmission must be upgraded (the 500KV backbone must be brought closer to the wind farm, along the Cuestecita-Valledupar-Copey corridor) to be able to take more wind energy.
- Wind uncertainty does trigger need for greater reserves.

- Financial viability of simple cycle gas turbines is in danger with wind deployment (it is known that they are already in trouble).
- But they are needed to survive El Niño phenomena, so they are compensated based on their capacity.
- The features that make the SuperOPF unique (treatment of uncertainty, endogenous locational reserve, flexible storage model) allows it to perform the study with a single, integrated piece of software.

Future work

- In the AC version, the central QP with the integer variables has turned out to require very long solution times for medium-sized problems, making it unsuitable for LR methods requiring repeated solutions, except for small cases.
- More experimentation with solvers must be done, as well as looking into adaptive piecewise linear approximations to the quadratic costs, but also…
- Must look at alternative ways of including the reactive component.

Outline

Summary of Focus of Past Year

- **MATPOWER** enhancements, new v5 release.
- **Support** for others using SuperOPF tools.
- **Simulation framework** for testing two-stage market structure and receding horizon.
- Multi-period SuperOPF with **Unit Commitment**, completing, refining, testing the 3rd generation code.
- Potential **expansion planning** directions.

Part II Overview: Generation Expansion Planning

- Which question do we want to answer?
- A formulation for generation expansion planning.
- Embellishments.
- Status and work to be done.
- Note: deterministic, but AC.

The question

- So I have just a few specific expansion projects and I want to choose to invest in even fewer; most practical case, … or
- I would like to see what would be the ideal expansion plan if I was free to place any kind of plant anywhere – useful for predicting how the energy matrix should change over time. Maybe not practical, but a question that must be asked nonetheless, especially in times of rapid transformation. This problem is very large.

For the first question…

- Other people in the group have come up with good formulations using DC flows, even including transmission expansion, and multi-stage decision making.
- For the second question: we'll go through some slides at a glance to define a model that can yield good quality solutions without claiming global optimality (these slides mostly for reference).

Cost

$$f(P_{\max}, P, Q, U, V, Z) = \sum_{t=1}^{n_t} \phi^t \sum_{i=1}^{n_g} \left[u^{ti} d^{ti} + v^{ti}\left(a^{ti} + b^{ti} p^i_{\max}\right)\right] - \sum_{i=1}^{n_g} u^{n_t,i} C^i_{RES}$$

$$+ \sum_{t=1}^{n_t} \phi^t \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \psi^{tk} C^{ti} p^{tik} + \sum_{t=1}^{n_t} \phi^t \sum_{i=1}^{n_g} z^{ti} C^i_{RET}$$

Here:

t: year or other time unit employed in the planning horizon, nt periods

k: scenario (from load duration curve histogram or another method)

i: plant index

v: investment decision

u: operation status in given year

z: retirement decision (fittingly, the last letter of the alphabet)

ϕ: discount factor

ψ: weight of scenario (hours/year, for example)

d: fixed yearly cost

(a, b): investment cost

$Cres$: salvage value at end of horizon

C: linear coefficient of operation cost

$Cret$: retirement cost

OPF constraints

$$g_P\left(\theta^{tk}, V^{tk}, S_P^{tk}\right) = 0, \quad t = 1, 2, \ldots n^t, \qquad k = 1, 2, \ldots n_k^t,$$
$$g_Q\left(\theta^{tk}, V^{tk}, S_Q^{tk}\right) = 0, \quad t = 1, 2, \ldots n^t, \qquad k = 1, 2, ..., n_k^t,$$
$$h\left(\theta^{tk}, V^{tk}\right) \leq 0, \quad t = 1, 2, \ldots n^t, \qquad k = 1, 2, \ldots n_k^t$$

$$\alpha^i p_{\max}^i u^{ti} \omega^{tik} \leq p^{tik} \leq p_{\max}^i u^{ti} \omega^{tik}$$
$$\beta_1^i p_{\max}^i u^{ti} \omega^{tik} \leq q^{tik} \leq \beta_2^i p_{\max}^i u^{ti} \omega^{tik}$$

Coordination constraints

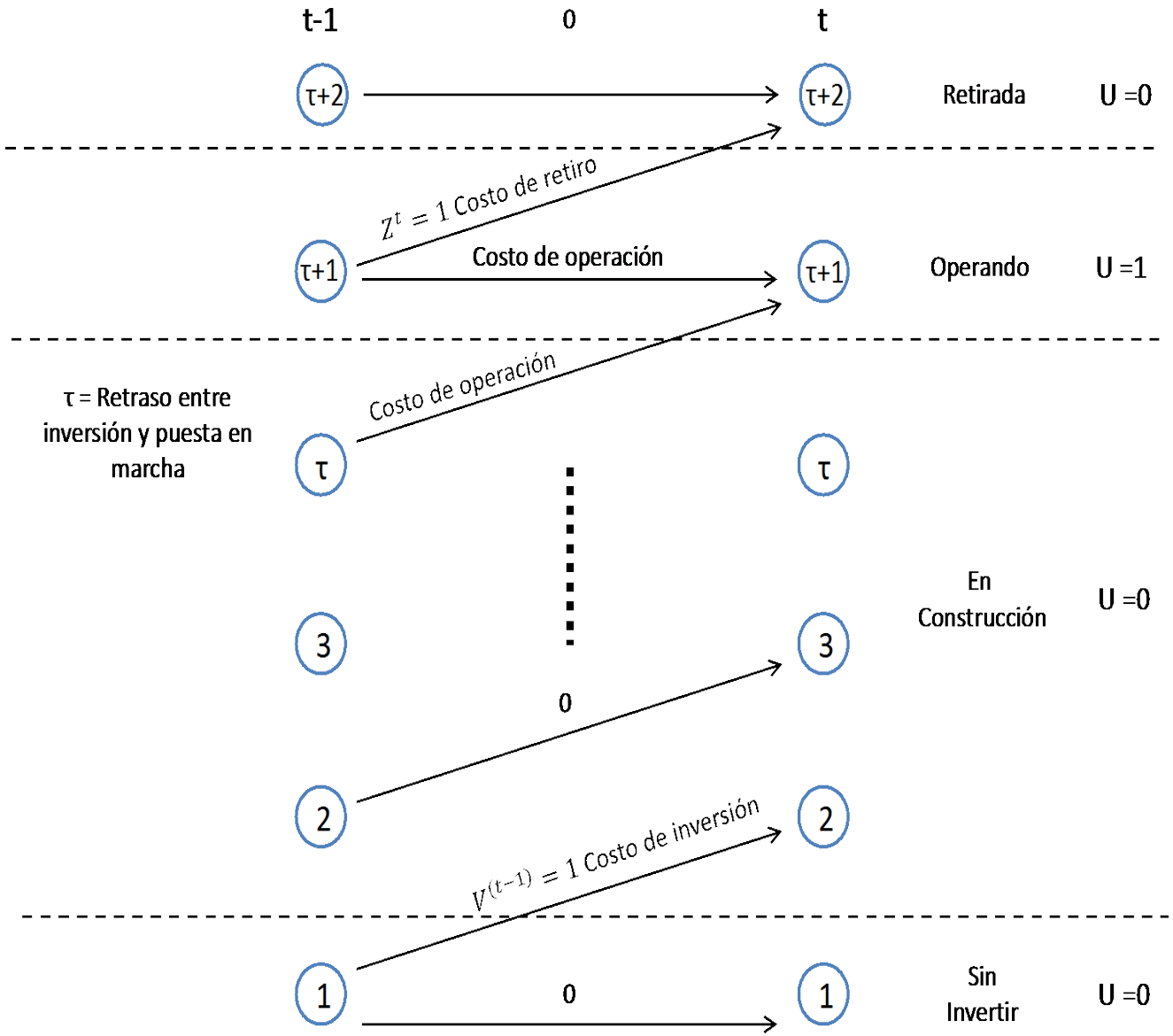
$$s_P^{tik} - u^{ti} w^{ti} p^{tik} = 0$$

$$s_Q^{tik} - u^{ti} w^{ti} q^{tik} = 0$$

$$R^{tkr} - \sum_{i \in I^{tkr}} \left(p_{\max}^{i} - p^{tik} \right) \leq 0$$

Other constraints…

- (U,V,Z) must meet transition constraints that are best cast in a DP transition graph:


t-1
0
t
τ+2
τ+2
Retirada
U =0
$Z^t = 1$ Costo de retiro
τ+1
Costo de operación
τ+1
Operando
U =1
τ = Retraso entre inversión y puesta en marcha
Costo de operación
τ
τ
3
3
En Construcción
U =0
0
2
2
$V^{(t-1)} = 1$ Costo de inversión
1
0
1
Sin Invertir
U =0

Lagrangian & friends

$$\mathcal{L}(P_{\max}, P, Q, U, V, Z, S_P, S_Q, \lambda_P, \lambda_Q, \mu) =$$

$$\sum_{t=1}^{n_t} \phi^t \sum_{i=1}^{n_g} \left[u^{ti} d^{ti} + v^{ti}\left(a^{ti} + b^{ti} p^i_{\max}\right)\right] - \sum_{i=1}^{n_g} u^{n_t,i} \mathrm{C}^i_{RES} + \sum_{t=1}^{n_t} \phi^t \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \psi^{tk} C^{ti} p^{tik}$$

$$+ \sum_{t=1}^{n_t} \phi^t \sum_{i=1}^{n_g} z^{ti} C^i_{RET} + \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \lambda_P^{tik}\left(s_P^{tik} - u^{ti}\omega^{ti} p^{tik}\right) + \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \lambda_Q^{tik}\left(s_Q^{tik} - u^{ti}\omega^{ti} q^{tik}\right)$$

$$+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{r=1}^{n_R^{tk}} \mu^{tkr}\left[R^{tkr} - \sum_{i \in I^{tkr}} \left(p^i_{\max} - p^{tik}\right)\right]$$

$$+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \left[\frac{b}{2}\left(s_P^{tik} - \hat{s}_P^{tik}\right)^2 + \frac{b}{2}\left(s_Q^{tik} - \hat{s}_Q^{tik}\right)^2 + \frac{b}{2}\left(p^{tik} - \hat{p}^{tik}\right)^2 + \frac{b}{2}\left(q^{tik} - \hat{q}^{tik}\right)^2\right]$$

$$+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \frac{c}{2}\left[\left(s_P^{tik} - u^{ti}\omega^{ti} p^{tik}\right)^2 + \left(s_Q^{tik} - u^{ti}\omega^{ti} q^{tik}\right)^2\right]$$

Linearizing non-separable terms

$$\mathcal{L}(P_{\max}, P, Q, U, V, Z, S_P, S_Q, \lambda_P, \lambda_Q, \mu) =$$

$$\sum_{t=1}^{n_t} \phi^t \sum_{i=1}^{n_g} \left[u^{ti} d^{ti} + v^{ti}\left(a^{ti} + b^{ti} p^i_{\max}\right)\right] - \sum_{i=1}^{n_g} u^{n_t,i} \mathrm{C}^i_{RES} + \sum_{t=1}^{n_t} \phi^t \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \psi^{tk} C^{ti} p^{tik}$$

$$+ \sum_{t=1}^{n_t} \phi^t \sum_{i=1}^{n_g} z^{ti} C^i_{RET} + \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \lambda_P^{tik}\left(s_P^{tik} - u^{ti}\omega^{ti} p^{tik}\right) + \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \lambda_Q^{tik}\left(s_Q^{tik} - u^{ti}\omega^{ti} q^{tik}\right)$$

$$+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{r=1}^{n_R^{tk}} \mu^{tkr}\left[R^{tkr} - \sum_{i \in I^{tkr}} \left(p^i_{\max} - p^{tik}\right)\right]$$

$$+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \left[\frac{b}{2}\left(s_P^{tik} - \hat{s}_P^{tik}\right)^2 + \frac{b}{2}\left(s_Q^{tik} - \hat{s}_Q^{tik}\right)^2 + \frac{b}{2}\left(p^{tik} - \hat{p}^{tik}\right)^2 + \frac{b}{2}\left(q^{tik} - \hat{q}^{tik}\right)^2\right]$$

$$+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \left[c\left(\hat{s}_P^{tik} - \hat{u}^{ti}\omega^{ti}\hat{p}^{tik}\right)\left(s_P^{tik} - u^{ti}\omega^{ti} p^{tik}\right) + c\left(\hat{s}_Q^{tik} - \hat{u}^{ti}\omega^{ti}\hat{q}^{tik}\right)\left(s_Q^{tik} - u^{ti}\omega^{ti} q^{tik}\right)\right]$$

- And, after defining the dual functional as a minimization of the Lagrangian for fixed duals, then reordering the summations and separating the inner minimizations, we come up with a dual functional whose evaluation amounts to minimizations over smaller sets of variables.

$$
\begin{aligned}
&q\left(\lambda_P, \lambda_Q, \mu\right) = \\
&\sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \min_{S_P^{tk}, S_Q^{tk}} \left\{ \sum_{i=1}^{n_g} \left\{ \begin{array}{l} \frac{b}{2}\left(s_P^{tik}\right)^2 + \left[\lambda_P^{tik} - b\left(\hat{s}_P^{tik}\right) + c\left(\hat{s}_P^{tik} - \hat{u}^{ti}\omega^{ti}\hat{p}^{tik}\right)\right] s_P^{tik} \\ + \frac{b}{2}\left(s_Q^{tik}\right)^2 + \left[\lambda_Q^{tik} - b\left(\hat{s}_Q^{tik}\right) + c\left(\hat{s}_Q^{tik} - \hat{u}^{ti}\omega^{ti}\hat{q}^{tik}\right)\right] s_Q^{tik} \end{array} \right\} \right\}
\end{aligned}
$$

Such that

$$
\begin{aligned}
g_P\left(\theta^{tk}, V^{tk}, {S_P}^{tk}\right) &= 0, \\
g_Q\left(\theta^{tk}, V^{tk}, {S_Q}^{tk}\right) &= 0, \\
h\left(\theta^{tk}, V^{tk}\right) &\leq 0,
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{i=1}^{n_g} \min_{P_{\max}^i, P^i, Q^i, U^i, V^i, Z^i} \left\{ -\left(u^{n_t,i} C_{RES}^i\right) + \sum_{t=1}^{n_t} \left\{ \begin{array}{l} \phi^t \left[\begin{array}{l} u^{ti} d^{ti} + v^{ti}\left\{a^{ti} + \left(b^{ti} p_{\max}^i\right)\right\} + \left(z^{ti} C_{RET}^i\right) \\ - \left(\sum_{k=1}^{n_k^t} \sum_{r=1}^{n_R^{tk}} I(\Im^{tkr}, i) \mu^{tkr} \right) p_{\max}^i \end{array} \right] \\ + \sum_{k=1}^{n_k^t} \left\{ \begin{array}{l} \frac{b}{2}\left(p^{tik}\right)^2 + \left[\begin{array}{l} -\lambda_P^{tik} u^{ti}\omega^{ti} + b\left(\hat{p}^{tik}\right) \\ -c\left(\hat{s}_P^{tik} - \hat{u}^{ti}\omega^{ti}\hat{p}^{tik}\right) \\ + \left(\phi^t \psi^{tk} C^{ti}\right) \\ + \sum_{r=1}^{n_R^{tk}} \Im(I^{tkr}, i) \mu^{tkr} \end{array} \right] p^{tik} \\ + \frac{b}{2}\left(q^{tik}\right)^2 + \left[\begin{array}{l} -\lambda_Q^{tik} u^{ti}\omega^{ti} + b\left(\hat{q}^{tik}\right) \\ -c\left(\hat{s}_P^{tik} - \hat{u}^{ti}\omega^{ti}\hat{p}^{tik}\right) \end{array} \right] q^{tik} \end{array} \right\} \end{array} \right\} \right\}
\end{aligned}
$$

Such that

$$
\begin{aligned}
\alpha^i p_{\max}^i u^{ti}\omega^{tik} &\leq p^{tik} \leq p_{\max}^i u^{ti}\omega^{tik} \\
\beta_1^i p_{\max}^i u^{ti}\omega^{tik} &\leq q^{tik} \leq \beta_2^i p_{\max}^i u^{ti}\omega^{tik}
\end{aligned}
$$

$$
+ \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{r=1}^{n_R^{tk}} \mu^{tkr} R^{tkr} + \sum_{t=1}^{n_t} \sum_{k=1}^{n_k^t} \sum_{i=1}^{n_g} \left\{ \frac{b}{2}\left(\hat{s}_P^{tik}\right)^2 + \frac{b}{2}\left(\hat{s}_Q^{tik}\right)^2 + \frac{b}{2}\left(\hat{p}^{tik}\right)^2 + \frac{b}{2}\left(\hat{q}^{tik}\right)^2 \right\}
$$

- The first set of minimizations amount to solving many OPFs (AC). The second set of minimizations (one per generator or project) amounts to solving n_g dynamic programs.
- High degree of separation, and parallelizable.

Algorithm

- Standard Uzawa-style; only have to be careful with the μ updates.
- AC flows: will also select projects that are required just for VAR compensation.
- Embellishments: if heat rates are linear and the fuel supply functions are linear, then the operation cost for, say the jth coal unit is

$$\left[c_1 + c_2 \sum_{i \in I_{COAL}} p^i \right] \cdot h^j \cdot p^j$$

Can then linearize cross terms at each iteration, just like the augmentation terms.

Status

- The linear fuel supply function feature has been implemented; generators belong to (possibly overlapping) “fuel zones” and compete for fuel in those zones in a given time horizon/season.
- Production costs can now include fixed cost dependent on Pmax of project.
- A test study for the Colombian system is undergoing calibration of the fuel supply function model.

To be done

- Add seasonal energy restrictions to hydro plants (currently it requires manually adjusting the hydro generation cost).
- Change scenarios from being “typical hours” to being “typical trajectories” to model ramping requirements.
- Further away: find a way to bring transmission expansion, uncertainty into the picture.

Drawbacks

- Not stochastic (though the weights could be thought of as probabilities).
- Even with the fuel supply elasticity correction, this is a one-off formulation, not staged. Uncertainty abounds: technology development, evolution of fuel supply, climate, economics and demographics, as well as geopolitical affairs.
- But what-if analysis is made possible.

Questions?