Essence of Structure Preserving (ESP) Network Reductions for Engineering and Economic Analysis

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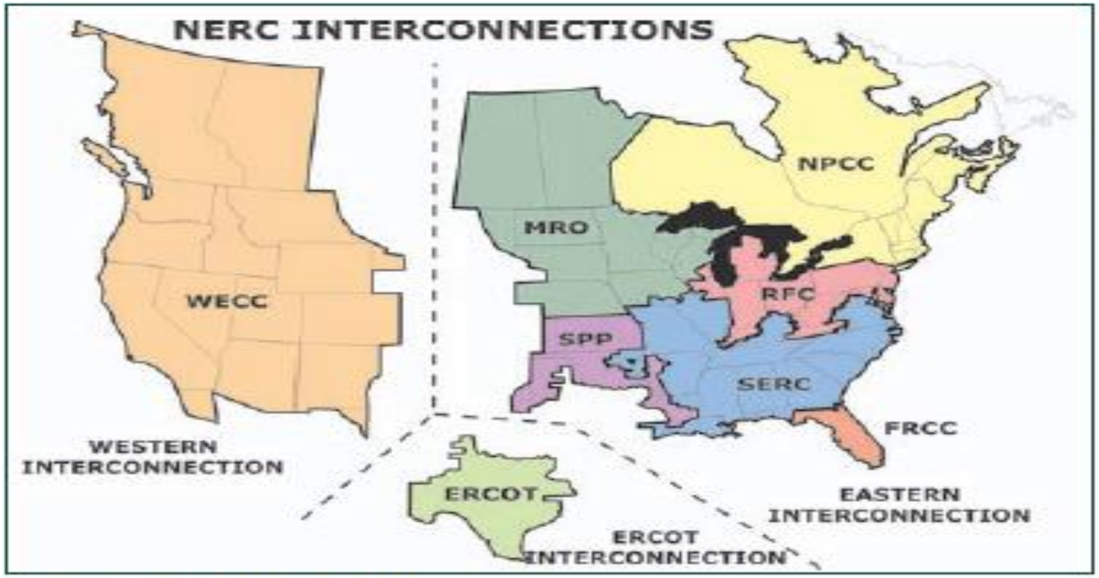
Overview

- Motivation-Develop network Eq's for US for generator investment studies
- Review (Modified) Ward-type network equivalent
  - Apply to EI, ERCOT & WECC
  - Model accuracy evaluation
    - Effect of dc model reduction
    - Effect of ac v. dc approximation
    - Base case, change cases & OPF
- Bus Aggregation--A different approach for reduction
  - Clustering
  - Equivalent branch reactances
  - Accuracy
- Conclusions

Study Objective

- Develop a backbone equivalent network model of the entire United States as a tool (to be made public domain) for the future grid engineering, market, environment studies.

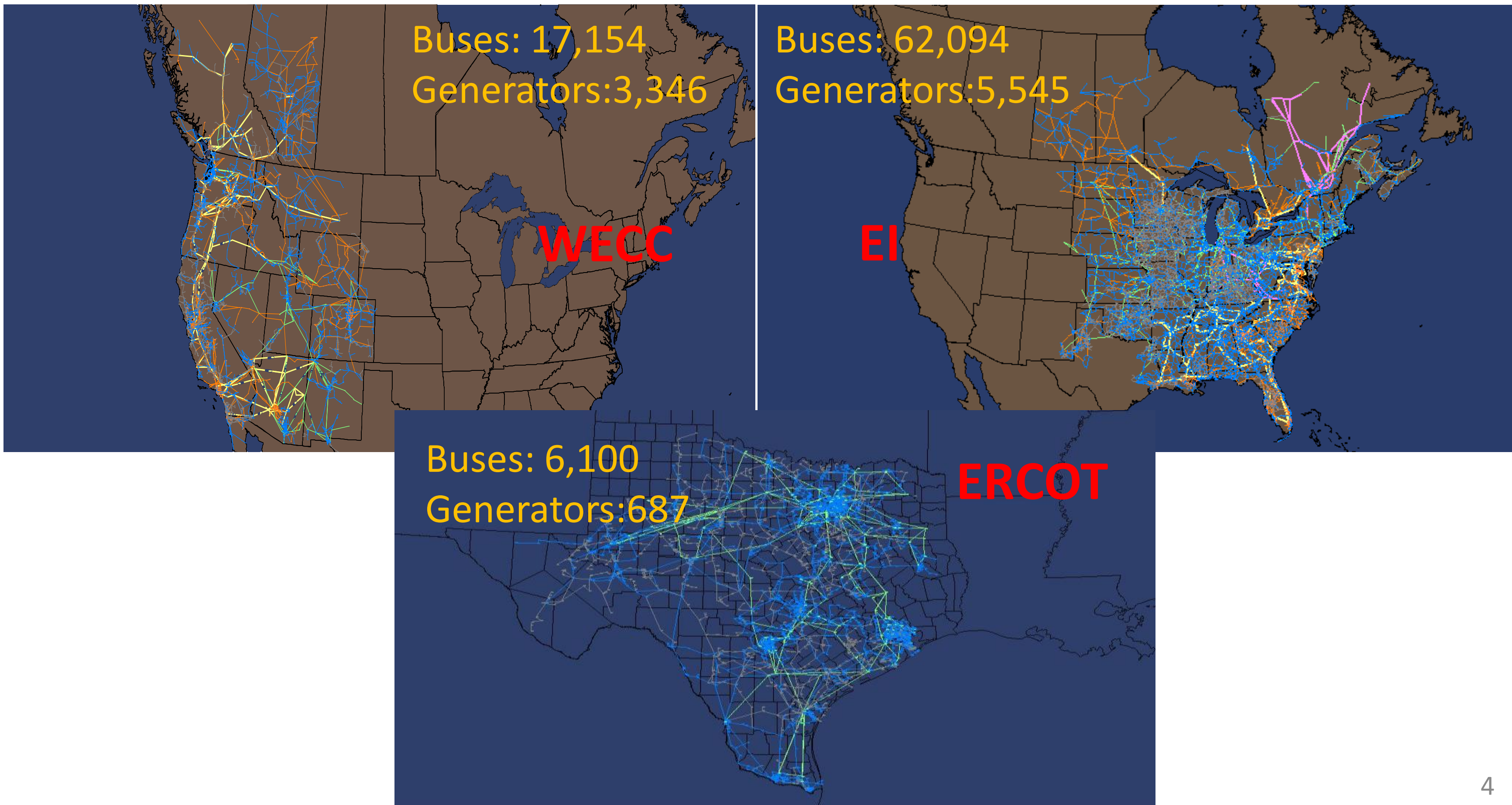
- Specific goals:
  - Accurately represent the WECC, EI, ERCOT network to match the base case
  - Reasonable accuracy for change cases
  - Reasonable accuracy for OPF-based studies



Source: NERC, 2006.

Data Base

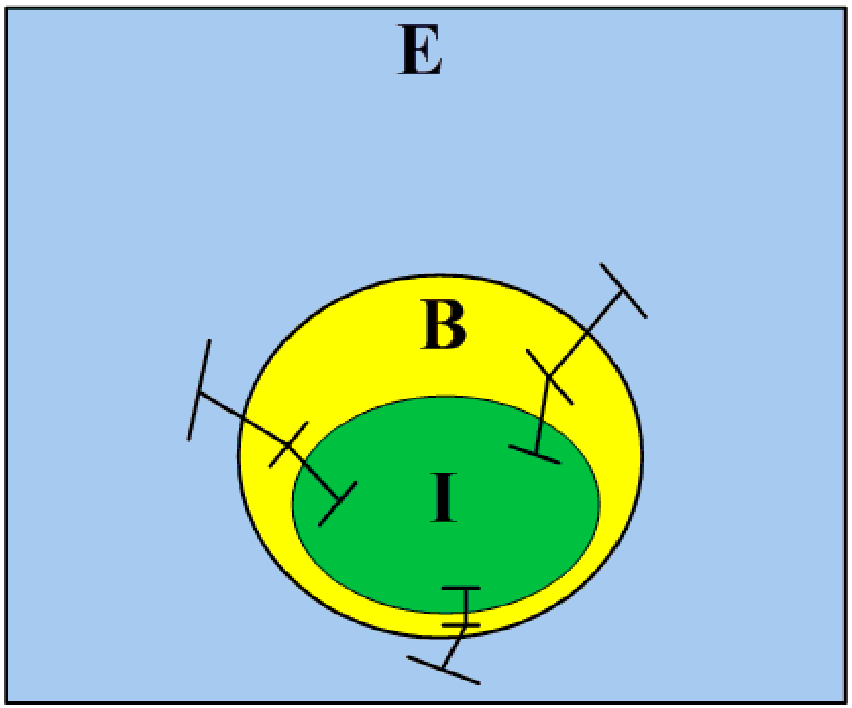
- EI, ERCOT, WECC data bases obtained from Energy Visuals (EV), Multiregional Modeling Working Group (MMWG) & ERCOT.
- Base case: year 2011 summer peak



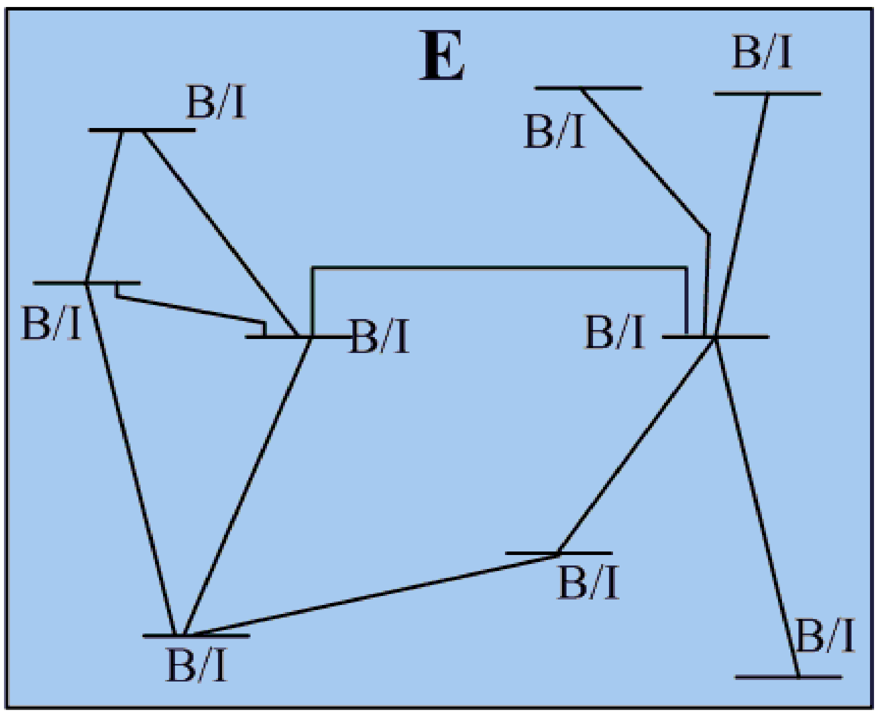
Limitation of Traditional Methods

- Traditionally, network equivalencing is performed by assuming linearity and eliminating (equivalencing) the unnecessary elements from the system.
- The system is divided into internal system (to retain), external system (to eliminate), and boundary buses.

E – External System I – Internal System B – Boundary Buses

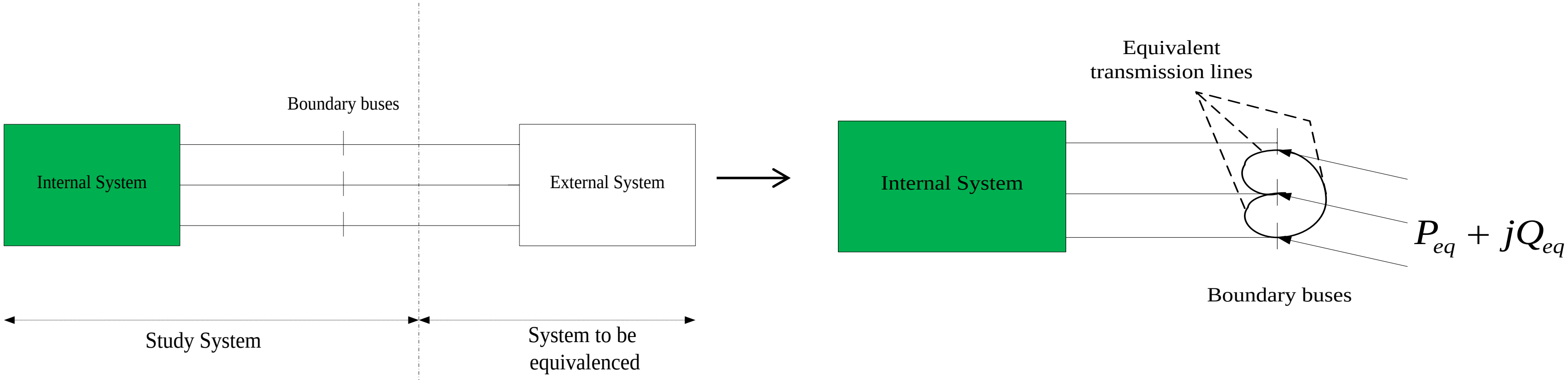


Traditional methods do...



In this study, we want...

Review-Ward Equivalent



- Ward-type equivalents "smear" the injections of external generators over a large number of boundary buses.
- For generator investment studies and GHG simulation tools it is impractical to model fractions of generators located at many buses.

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Solution-The Modified Ward Equivalent

- In this work, the traditional Ward equivalencing method is modified to generate a modified-Ward equivalent for the EI, ERCOT and WECC.
- In particular, the methodology employed includes:
  – use the network model generated by Ward's method (Gaussian elimination on the system admittance matrix)
  – move whole generators to 'retained buses' based on electrical distance
  – move load to compensate the movement of generators

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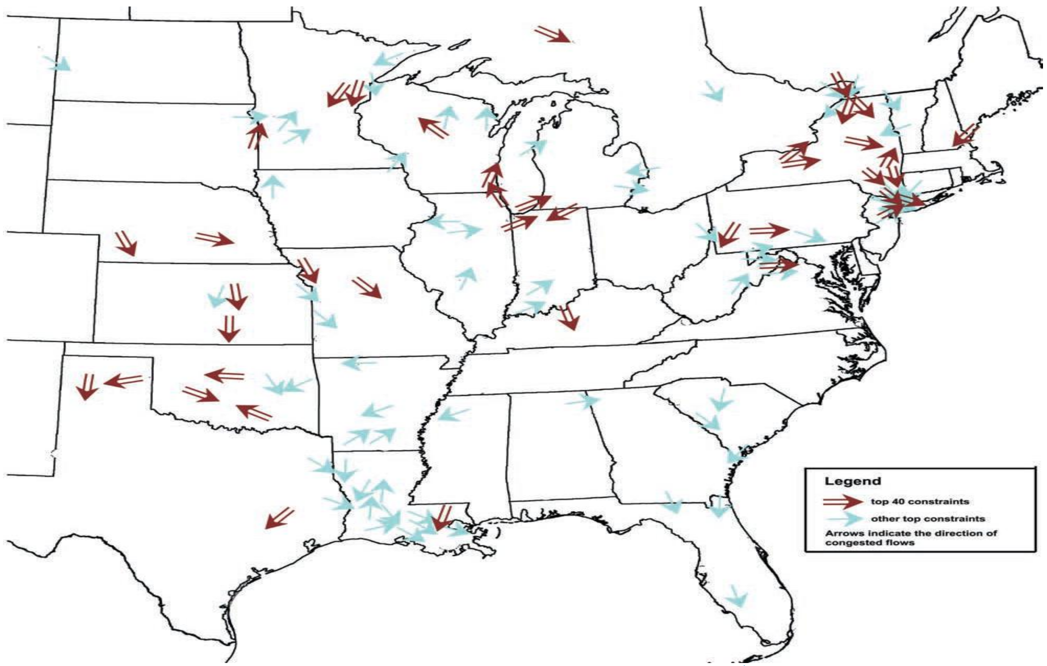
Selecting Elements to Retain

- Our objective is to retain critical congested transmission paths, congested areas, and as many of the high voltage buses as possible in the EI system.
- Historically congested transmission paths & areas are identified in the National Electric Transmission Congestion Study[1,2] commissioned by DOE and future congested transmission paths & areas are predicted in the



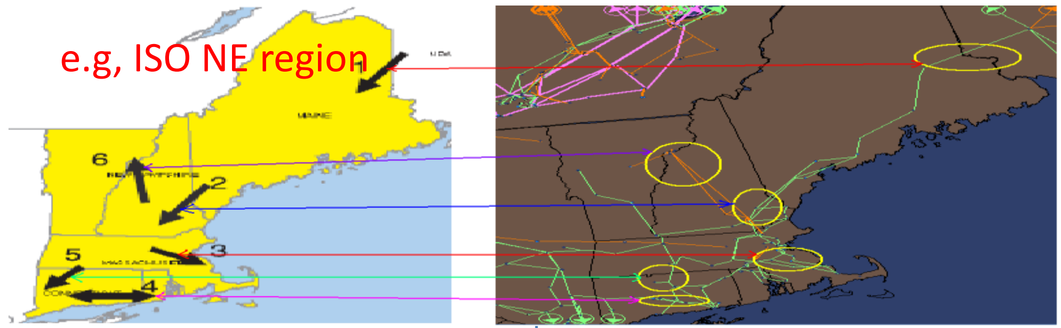
Most congested paths in EI – simulation[1]

1: http://www.oe.energy.gov/DocumentsandMedia/Congestion_Study_2006-9MB.pdf - 2006 version
2: http://congestion09.anl.gov/documents/docs/Congestion_Study_2009.pdf - 2009 version

Matching Between the Congestion Studies and the MMWG Data Base

- **Selection of lines and corresponding buses to retain was a 3-step procedure:**

Step 1: candidate selection



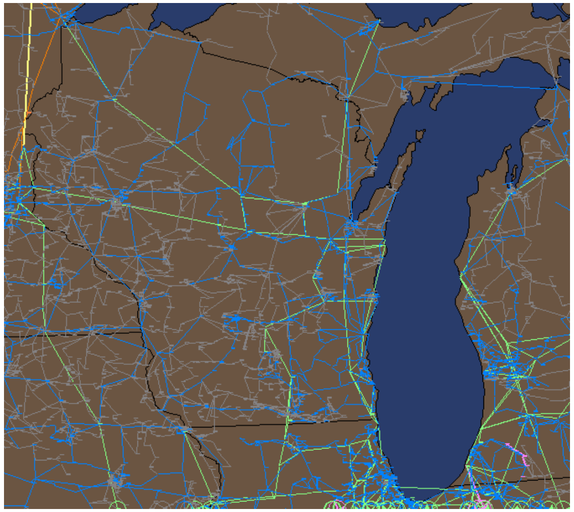
Step 2: validation through industry contacts

Company/Organization	Name of person contacted
AEP	Navin Bhatt
FRCC	Vince Ordax
ISO New England	Xiaochuan Luo, Eugene Litvinov
Midwest ISO	Rao Konidena, Mark Westendorf, Ryan H. Westphal, Loren Mayer
New York ISO	Michael Swider, Steve Corey
PJM	Mahendra Patel
RFC	John Idzior
SERC	Joe Spencer
Southern Company	Doug McLaughlin, Wyne Gambe
SPP	Mak Nagle
TVA	Dejim Lowe

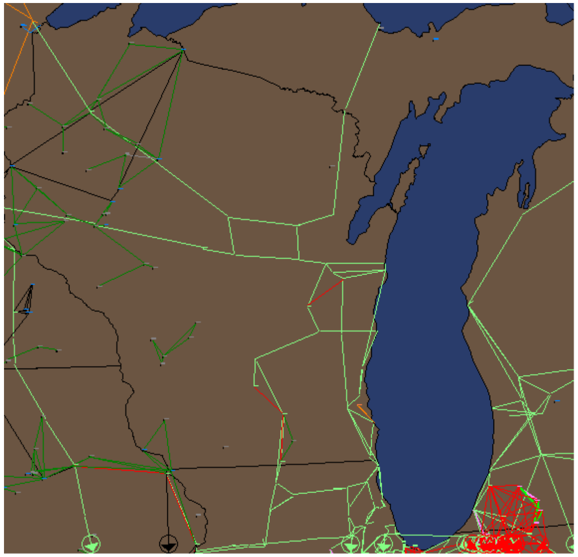
Step 3: modification based on feedback

Network Reduction Procedures

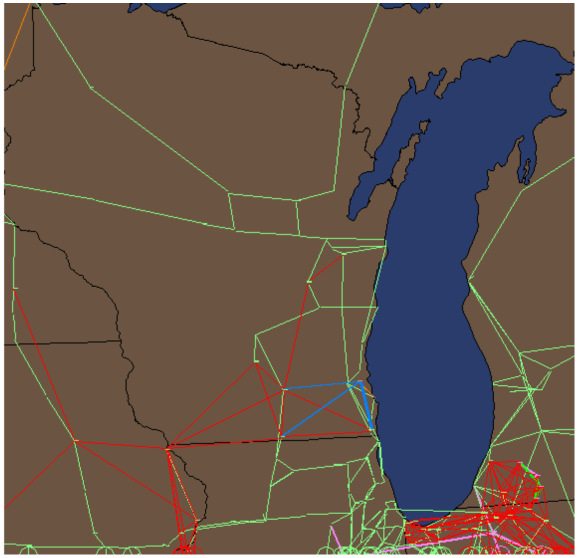
- 1. Apply the classical Ward equivalencing technique to remove all but the selected buses.
- 2. Apply the classical Ward equivalencing technique but retain both the selected buses and external generator buses. This model is used to determine the location of the generators in the final equivalent.
- 3. Apply the classical Ward equivalencing technique again to remove the external system. This step generates the network model for the final equivalent.



(b) Original WI system



(b) after step 2



(c) after step 3

Power system of Wisconsin

The Shortest Path Problem

- 4. **Move** generators to the boundary buses which are closest to the generators.
  - Based on graph theory, the generator moving problem can be formulated as the shortest weighted path problem.
  - Find a path between each generator bus and an internal bus such that the sum of the weights of constituent branches is minimized.
  - This shortest weighted path problem was solved using Dijkstra's algorithm [3, 4].
  - Other ways to measure electrical distance.

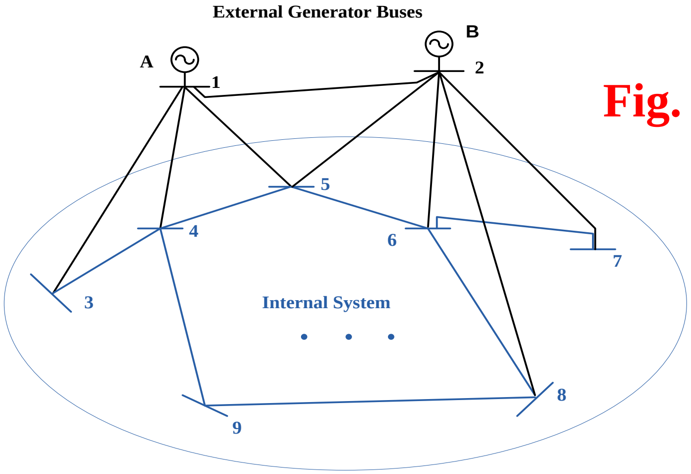


Fig. 1

generalization

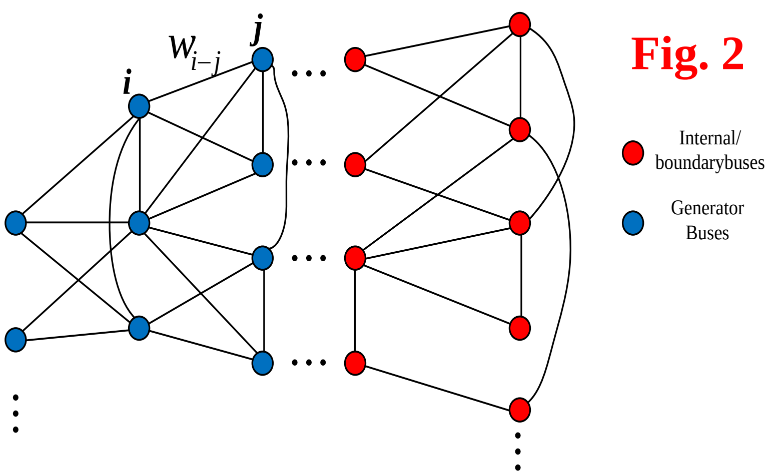
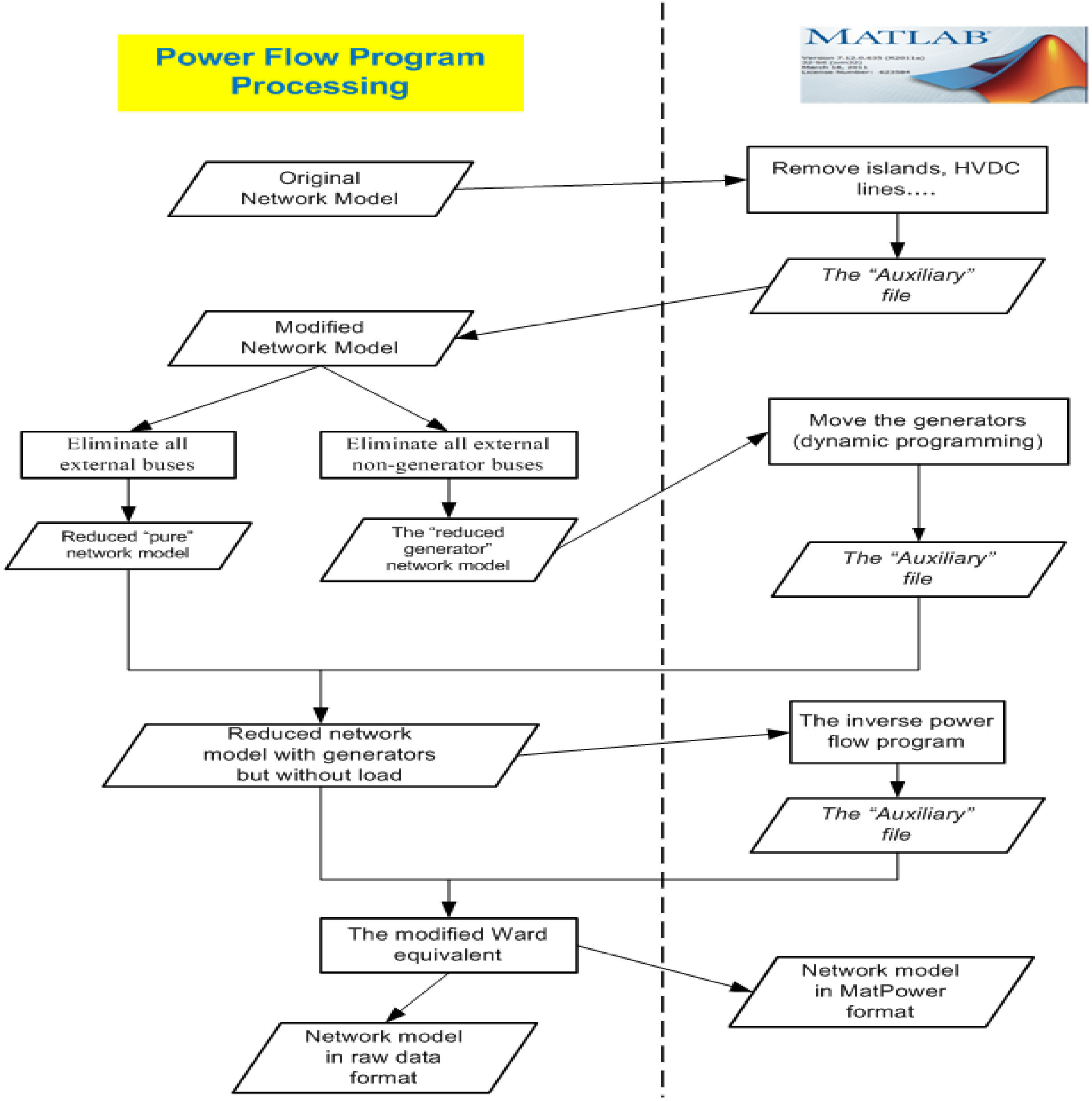


Fig. 2

3. E. W. Dijkstra, "A Note on Two Problems in Connection with Graphs," *Numerische Math*, vol. 1, no. 1, pp. 269-271, 1959.
4. B. V. Cherkassky, A. V. Andrew and T. Radzik, "Shortest Paths Algorithms: Theory and Experimental Evaluation", Mathematical Programming, Ser. A 73(2), pp. 129-174, 1996.

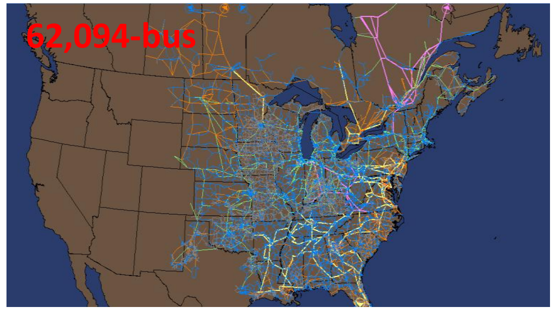
- 5. Move the load.
  - In the classical Ward equivalencing, external generation and load are broken up into fractions.
  - In the modified Ward method, move generators integrally.
  - Move load to match the base case using Inverse Power Flow.
  - Moving loads for gen investment /environmental studies in Cornell's SuperOPF not as critical since they can be scaled up/down in unison by region.
- 6. The slack bus is selected which offers the best numerical convergence during the reduction process.

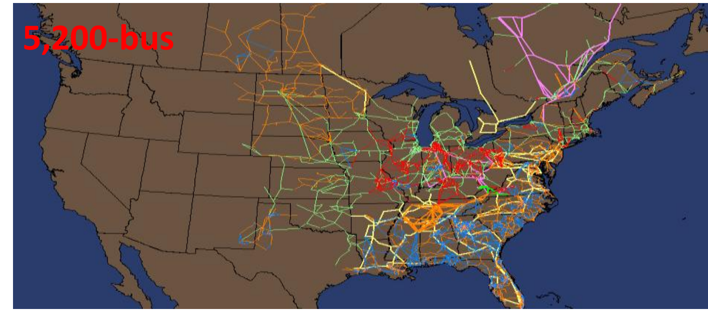
The Whole Process

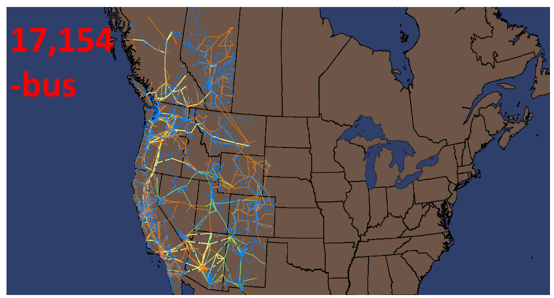


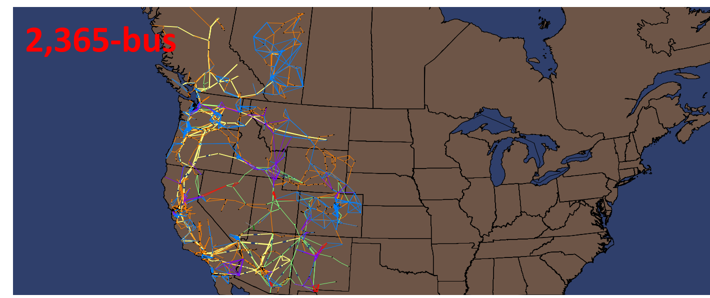
Equivalents of the Entire United States

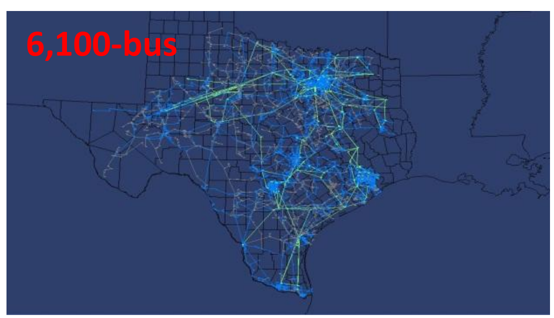
The 3 reduced models are interconnected through HVDC lines

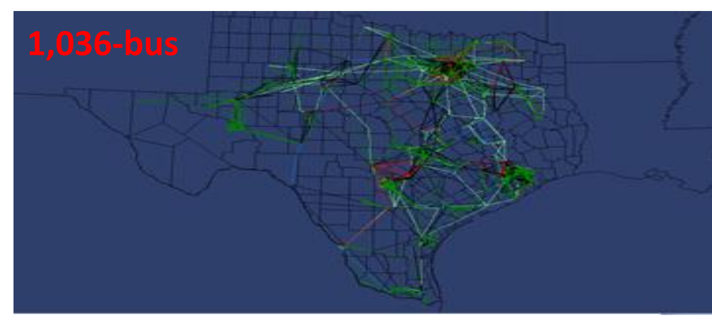












Full model: 85,348 buses (in total)

Reduced model: 8,601 buses (in total)

Model Evaluation Approach

- Error cause by dc-to-dc model reduction
  - EI, ERCOT, WECC
  - Test 1: Base Case + Changed Case
    - Re-dispatched a percentage of the coal fired generation to gas-fired units to mimic cap-and-trade scheme.
    - Metric: line-flow
  - Test 2: OPF Solution
    - Metric: LMP and production cost.
- Error caused in full model by ac-to-dc approximation
  - EI
  - Loss compensation
    - None
    - Single Load Multiplier
    - Zonal Multiplier

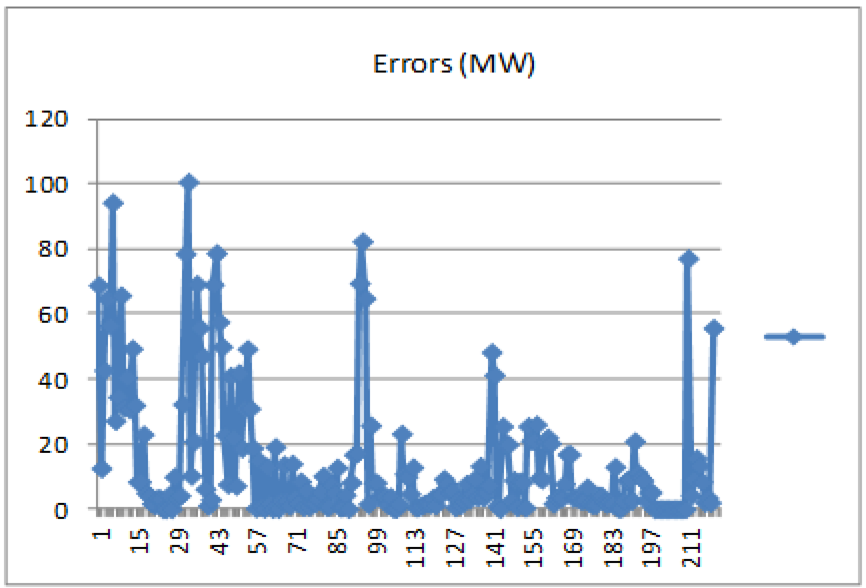
Model Evaluation Test-273 Bus dc EI

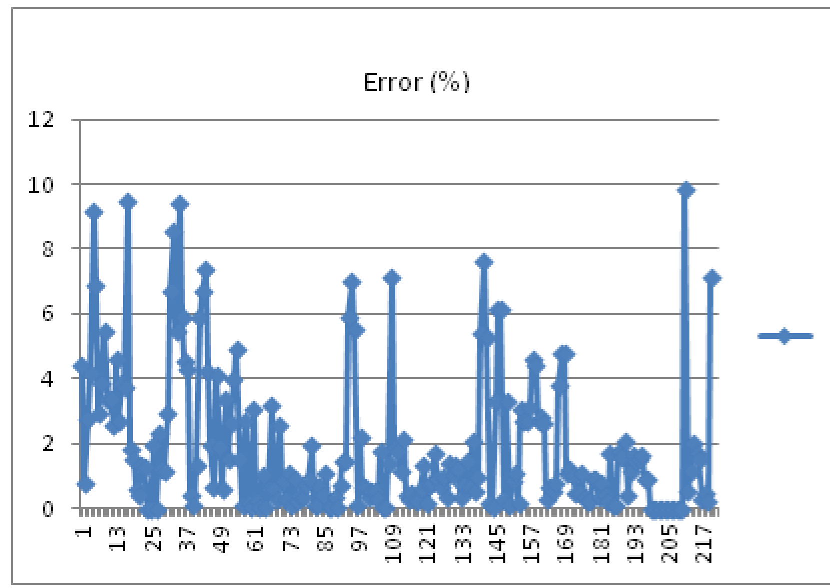
- Under the changed generation pattern, we
  - solve the dc power flow for the full model
  - solve the dc power flow for the reduced model
  - compare the two power flow solutions and calculate the errors:

$$Error_i = \left| Pf_i^{full} - Pf_i^{reduced} \right| \qquad Error_i\,\% = \frac{\left| Pf_i^{full} - Pf_i^{reduced} \right|}{(Lim_{MVA})_i}$$

where $(Lim_{MVA})_i$ represents the MVA rating of the retained line *i*.

- The test result for 1% decrease in coal generation is shown below

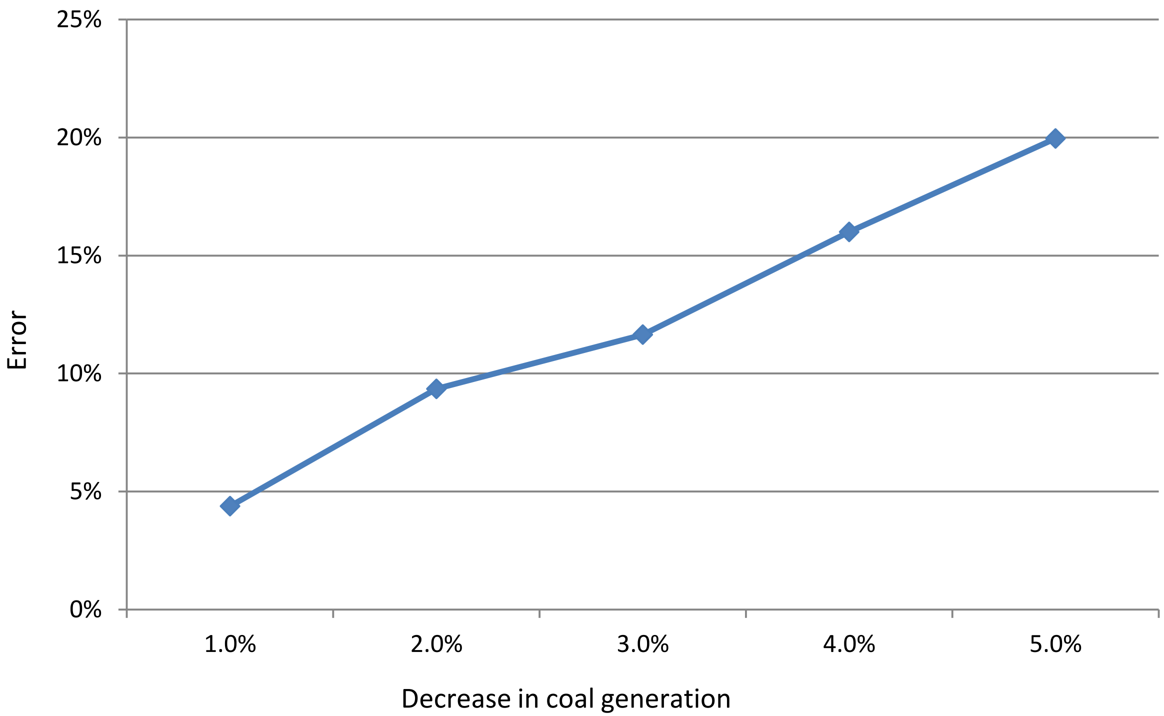




When the coal generation decreases by 1%, the largest error in the power flows on retained TL's reaches 100 MW, or 9.7%.

Model Evaluation Test-273 Bus EI

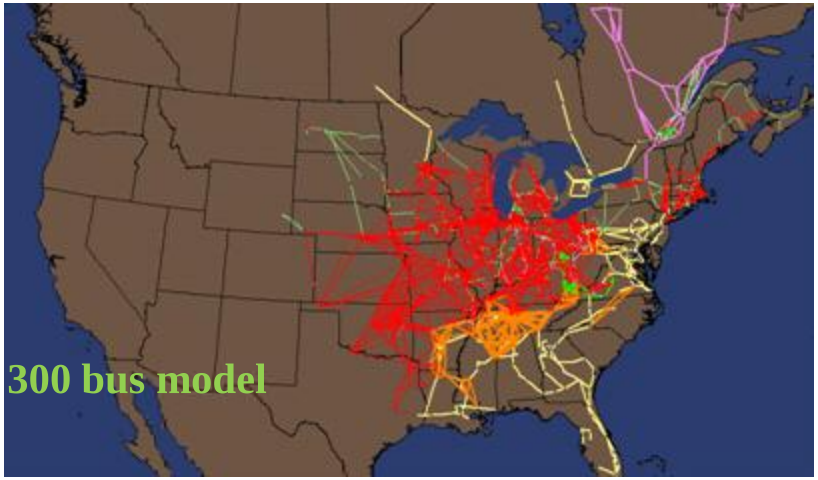
- Average percent error: $$Error_{Avg}\,\% = \frac{\sum_{i=1}^{N} \left| Pf_i^{full} - Pf_i^{reduced} \right| / (Lim_{MVA})_i}{N}$$
- For the 1% change in the generation pattern, the average errors in the line flows are calculated to be: 4.375%.
- Average error as a function of decrease in the coal generation:

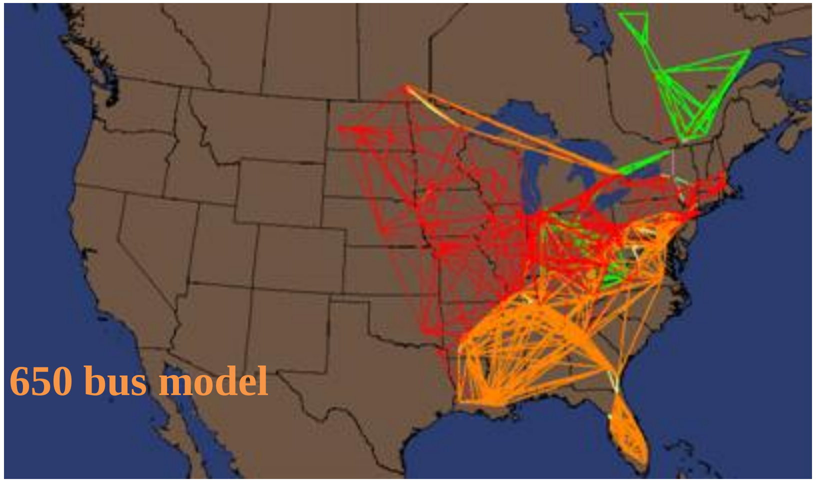


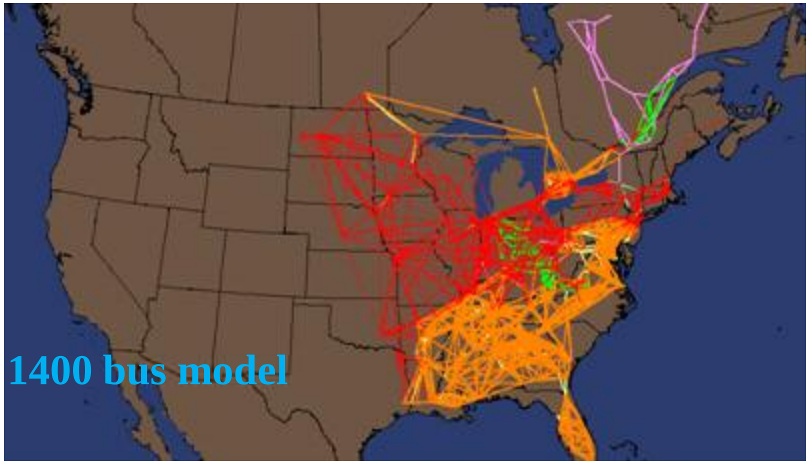
The reduced model PF solutions are becoming less accurate as the change in the generation pattern increases.

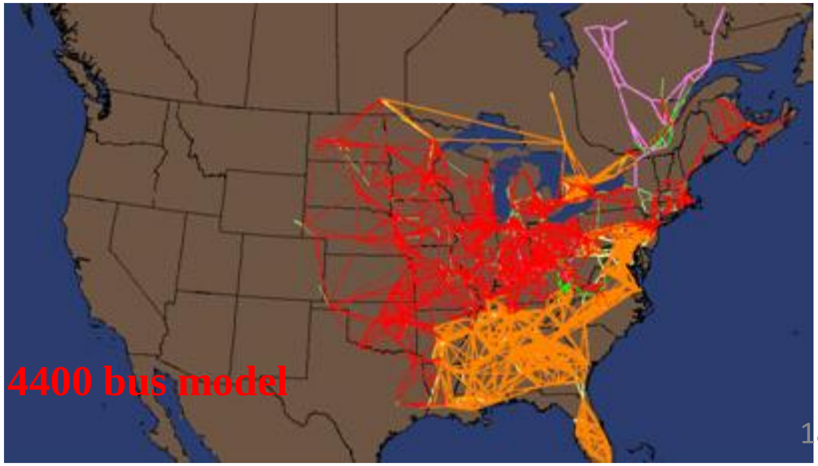
Accuracy vs. Size -EI

- The modified Ward equivalents generated are:
  - 300 bus model: retain all the congested lines based on DOE congestion study
  - 650 bus model: retain congested lines + 500kV and above buses
  - 1400 bus model: retain congested lines+345kVand above buses
  - 2800 bus model: retain congested lines+345kVand above buses+ part of 230kv buses
  - 4400 bus model: retain congested lines+230kV and above buses





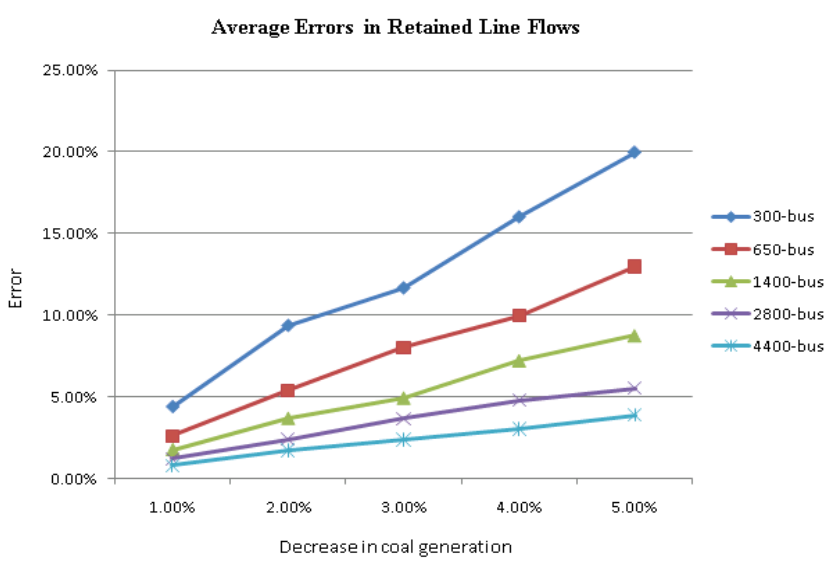




Accuracy vs. Size-EI

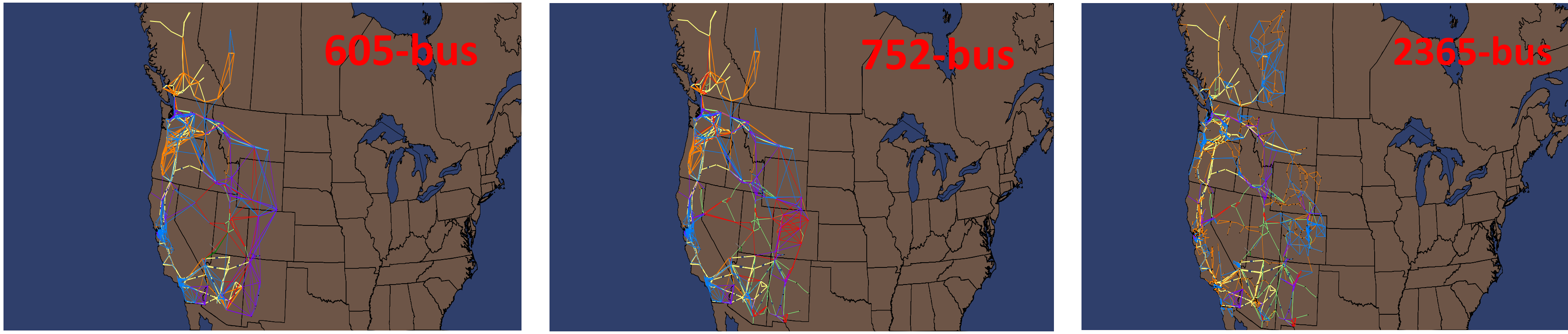
- The simulation results for these reduced models under different generation patterns are shown below:

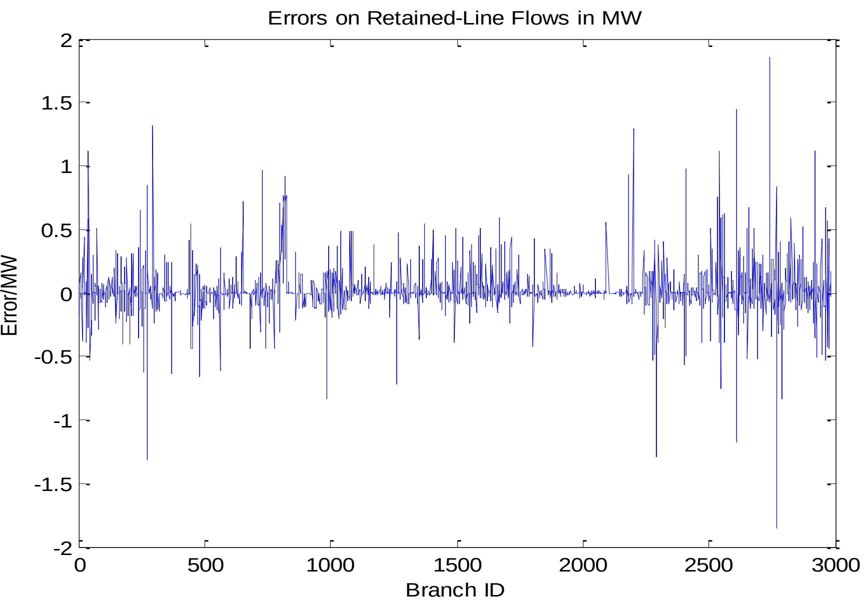
Bus # \ Decrease in coal	1%	2%	3%	4%	5%
300	4.38	9.35	11.65	16	19.96
650	2.71	5.38	8.01	9.96	12.97
1400	1.75	3.68	4.93	7.23	8.77
2800	1.21	2.40	3.67	4.79	5.53
4400	0.79	1.73	2.38	3.05	3.88



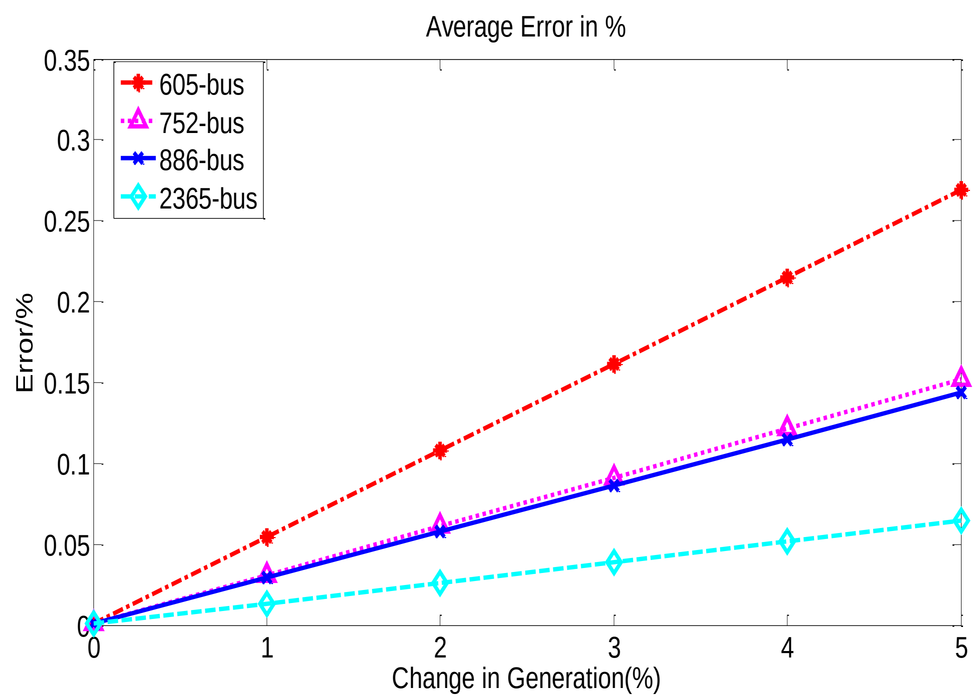
- A 2800 to 4400 bus model will be acceptable for the EI.

Model Evaluation Tests –WECC



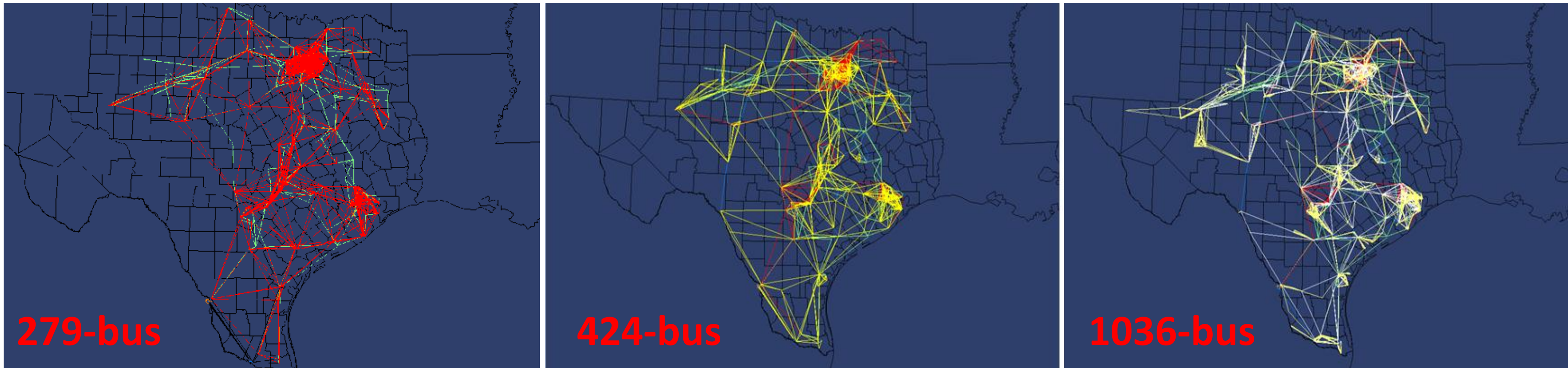


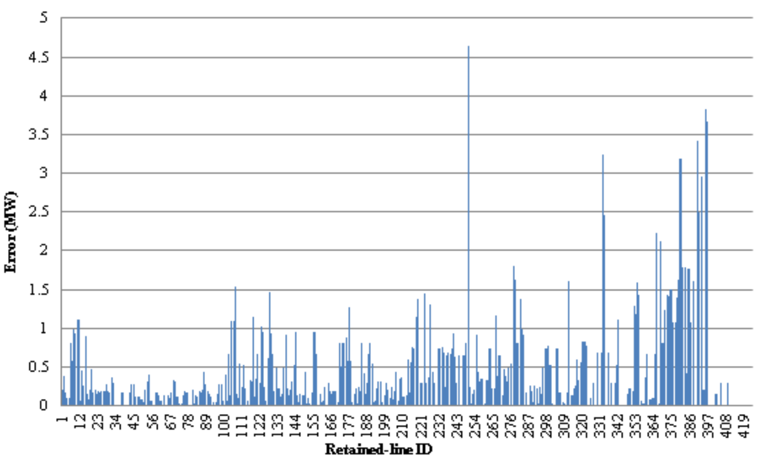
Error (MW) on retained lines when dispatch changes by 1% (2,365 bus model.)



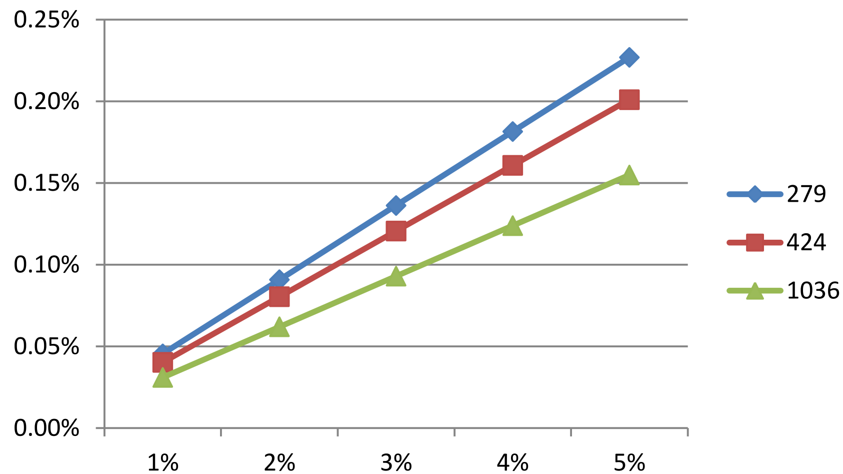
Avg. Error (%) vs. dispatch changes (size)

Model Evaluation Test - ERCOT





Error (MW) on retained lines when dispatch changes by 1% (279 bus model.)



Avg. Error (%) vs. dispatch changes (size)

Model Evaluation-dc OPF EI-5,200-bus equivalent

- Reduce 62,000 buses to 5,200 buses
- Computational efficiency for running dc vs. ac OPF: **Speedup factor of 16.5**.
- Production costs from the two models (16,244,321 $/Hour in the full system vs. 16,193,495.27 $/Hour in the equivalent.): **Error of 0.31%.**
- The average LMP differ by 0.0129 $/MWh,: **Error of 0.0254%**.
- The worst LMP difference 1.6841 ($/MWh): **Error 3.3% of the avg. LMP**.
- **The 5,200-bus modified Ward equivalent gives satisfactory results.**

Model Evaluation dc OPF
ERCOT 424-Bus Model

TABLE I.
COMPARISON BETWEEN THE DC OPF SOLUTIONS OF THE FULL AND 424-BUS-EQUIVALENT ERCOT MODELS

	Full Model	424-bus Equivalent	|Errors (MW)|	|Errors (%)|
Convergence	Y	Y	NA	NA
Total Cost ($/Hour)	1,363,111	1,360,559	2552	0.19%
Average LMP ($/MWh)	25.6163	25.6337	0.0174	0.068%

From Table I, it can be seen that-
-The error in the total operating costs 0.19%.

-The average LMPs differed by 0.0174 $/MWh: Error of 0.068%.

TABLE II
COMPARISON OF THE GENERATOR DISPATCH BETWEEN THE FULL AND 424-BUS-EQUIVALENT MODELS BASED ON DC OPF SOLUTIONS

Fuel Type	Equivalent (MW)	Full System (MW)	|Errors (MW)|	|Errors (%)|
nuclear	5131	5131	0.0	0.0%
coal	19576	19577	1.0	0.005%
natural gas	26041	25952	89.0	0.342%
wind	9380	9468	88.0	0.949%
Distillate Fuel Oil (Diesel,FO1,FO2,FO4)	-2	-2	0.0	0.000%
hydro	0	0	0.0	0.000%
waste heat	14	14	0.0	0.000%
wood or wood waste	50	50	0.0	0.000%
unknown	568	568	0.0	0.000%

Maximum error in generator dispatch is about1%.

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Model Evaluation Approach

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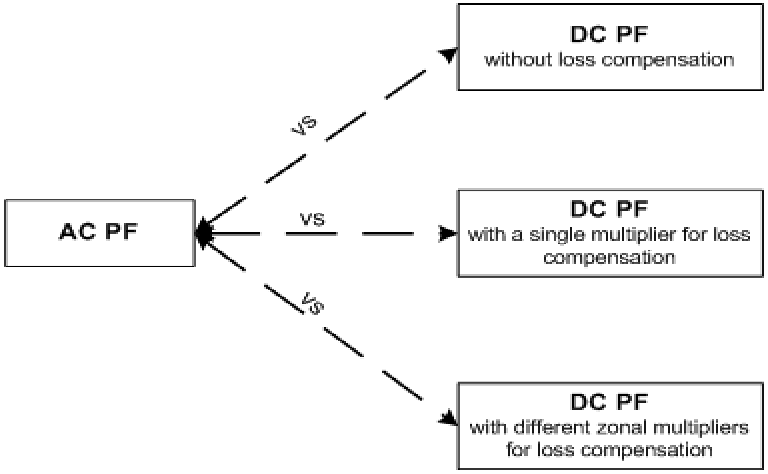
- Error cause by dc-to-dc model reduction
  - EI, ERCOT, WECC
  - Test 1: Base Case + Changed Case
    - Re-dispatched a percentage of the coal fired generation to gas-fired units to mimic cap-and-trade scheme.
    - Metric: line-flow
  - Test 2: OPF Solution
    - Metric: LMP and production cost.
- Error caused by ac-to-dc approximation
  - EI
  - Loss compensation
    - None
    - Single Load Multiplier
    - Zonal Multiplier

Errors: dc v. ac
Power Flow Approximation-Full EI Model

- ac power flow vs. dc power flow.
- For the dc PF model, we look at three cases:
  - No loss compensation
  - Losses compensated through single load multiplying
  - Losses compensated using “zonal” loss multipliers
- Metric: line flows

Cases considered:



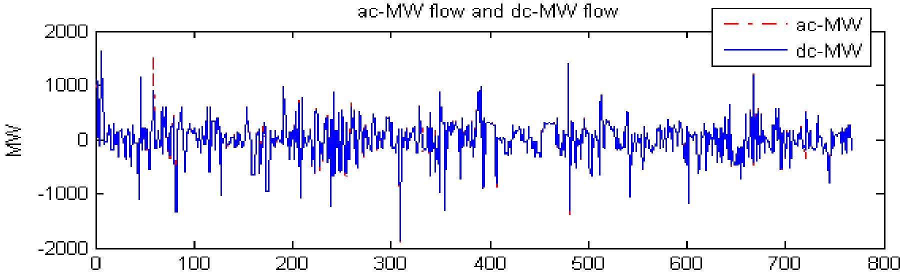
Reporting Line Flow Error Results

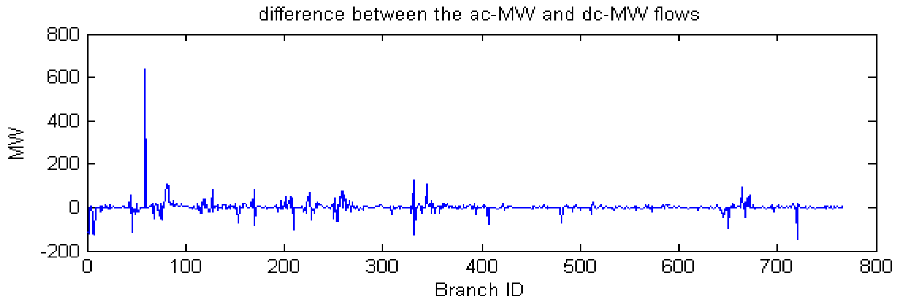
- Followed the same testing methods as discussed in Ref [A]. In reporting line-flow errors, we:
  - Neglect all lines with flows below 50MW
  - Neglect all lines that are loaded below 70% of the rating
  - Neglect all lines that have no MVA rating

- In addition, we neglected all branches that are 100kV and below. (We assume that for generation-investment studies, the MVA violations on lines that are 100kV and below can be corrected through long-term system planning.)

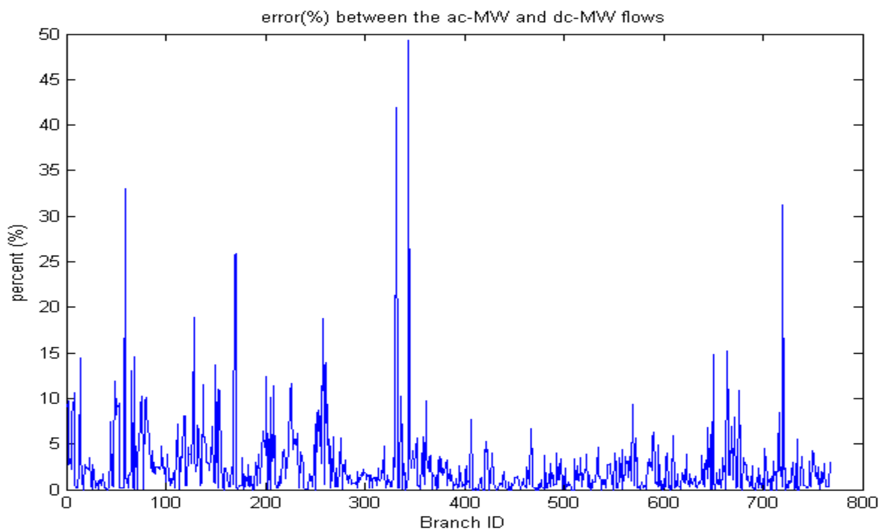
[A]B. Stott, J. Jardim, O. Alsaç, "DC Power Flow Revisited," *IEEE Trans. Power Syst.*, vol. 24, no. 3, Aug. 2009.

dc Line Flows with No Loss Compensation Full EI ac v. dc



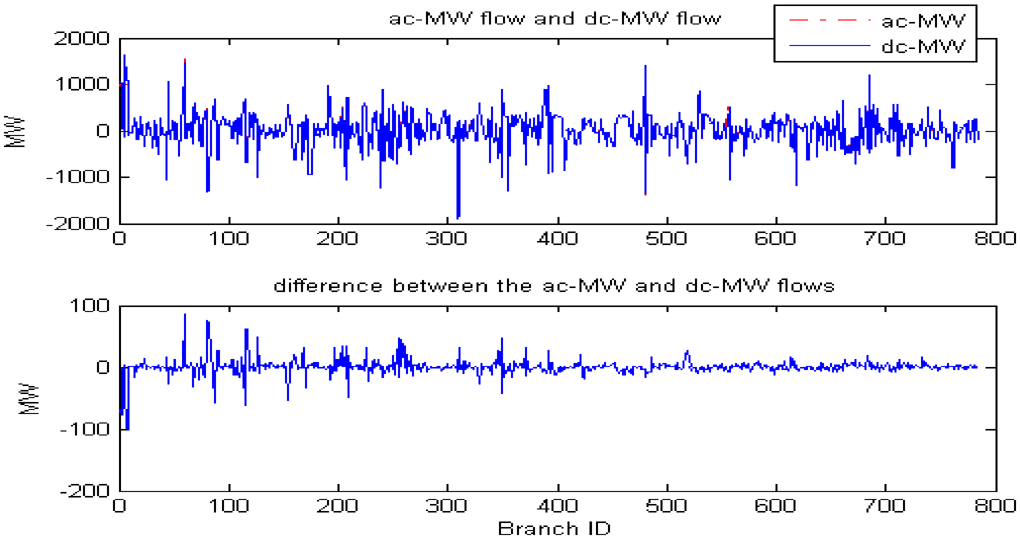


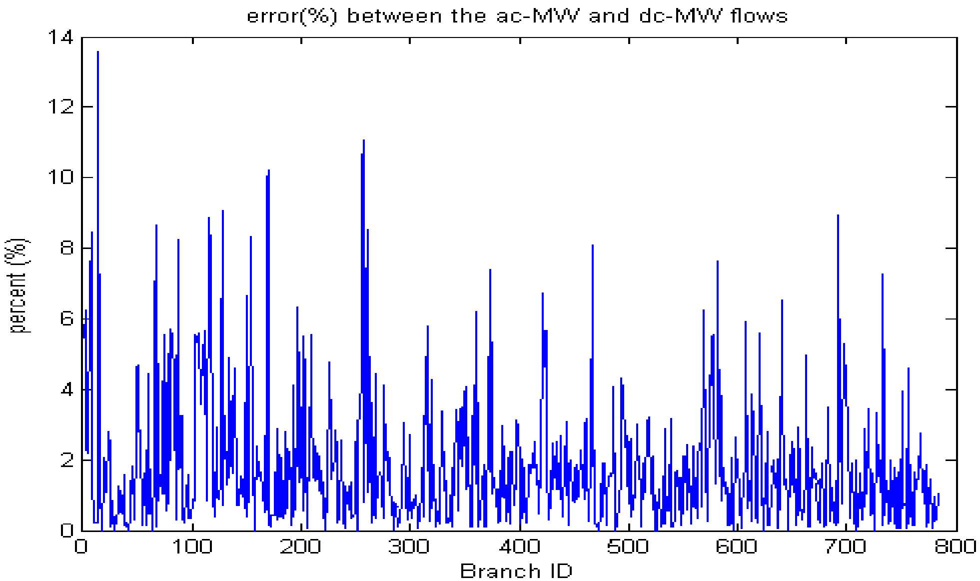
- The average error avg(|error|) is 10.91 MW
- The maximum error is about 600 MW



- Average error in percent is 2.54%
- Maximum error in percent is 49.33%

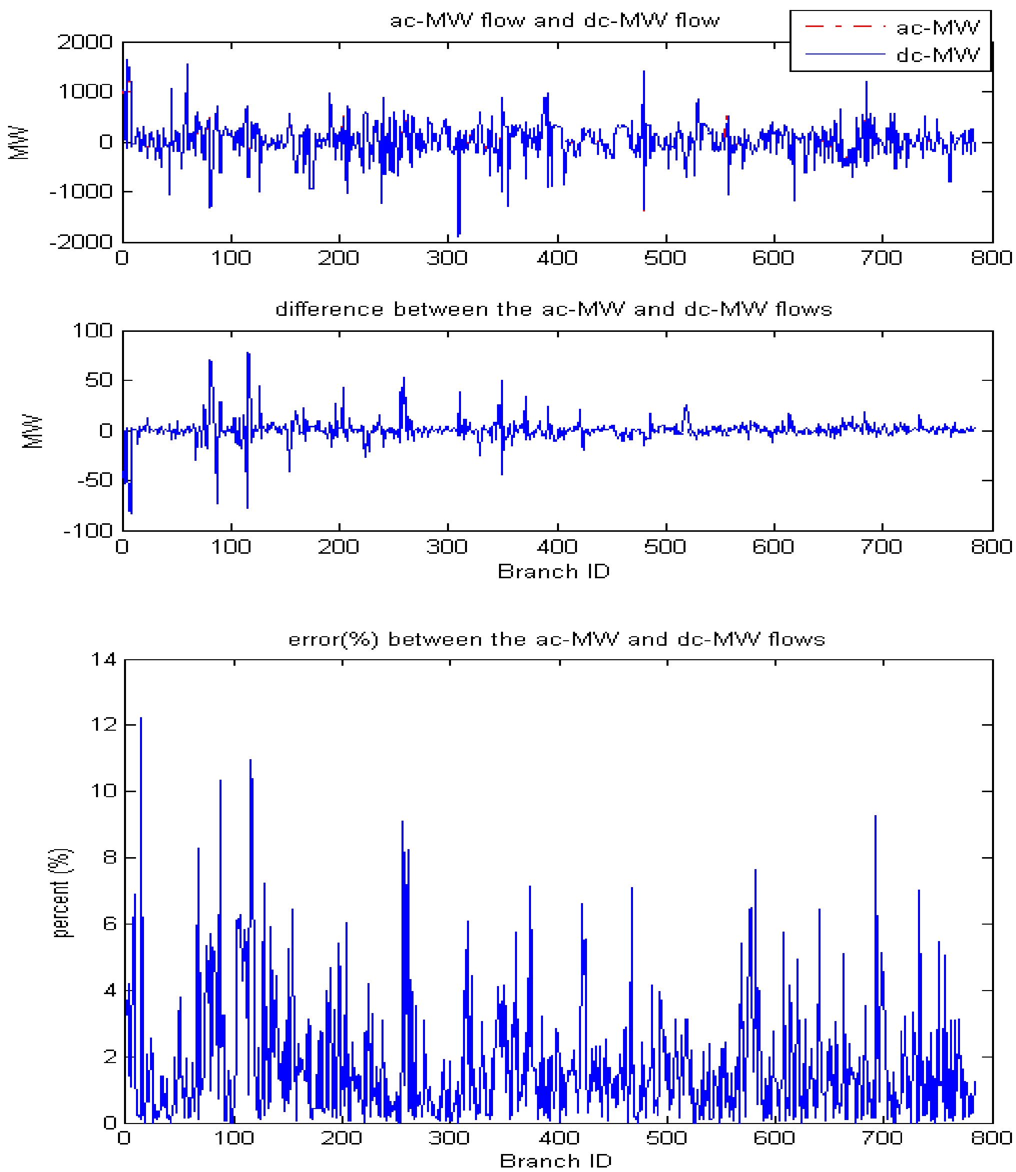
dc Line Flows with Single Loss Multiplier Full EI ac v. dc





Max error: 102MW
Max error in percent: 13.56%
Average error: 6.42MW
Average error in percent: 1.79%

dc Line Flows with Zonal Loss Multipliers Full EI ac v. dc



Max error: 83MW
Max error in percent: 12.20%
Average error: 5.62MW
Average error in percent: 1.64%

Improvement in Accuracy Using Loss Compensation Full EI ac v. dc

	DC PF with no loss compensation	DC PF with single scaling factor	DC PF with different scaling factors
Max error	600MW	102 MW	83 MW
Max error (%)	49.33%	13.56%	12.20%
Avg. error	10.91 MW	6.42 MW	5.62 MW
Avg. error (%)	2.54%	1.79 %	1.64%

Conclusions:

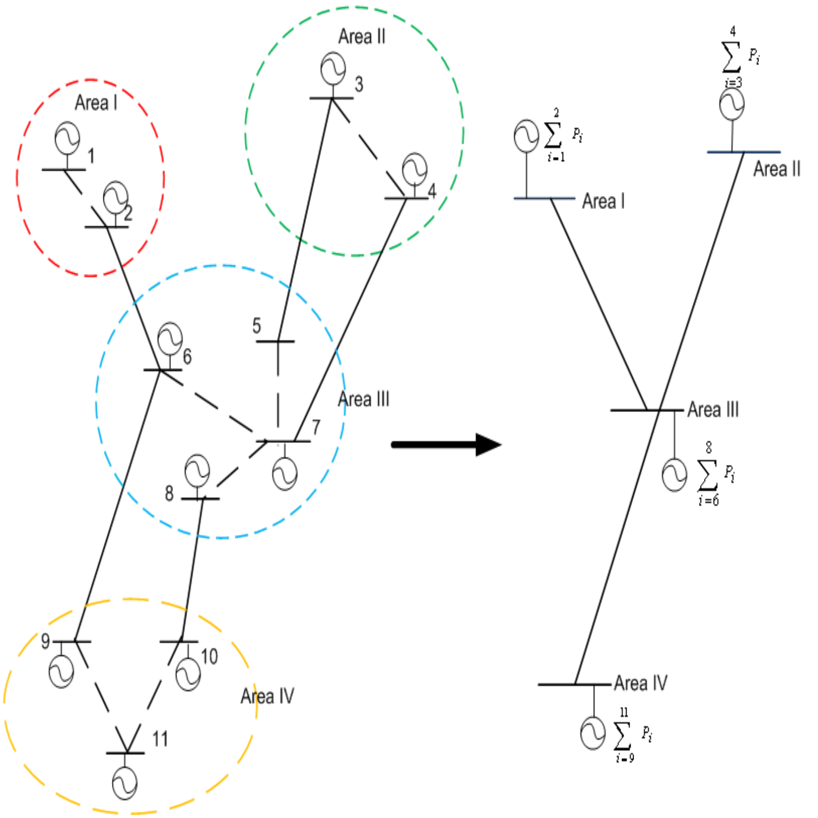
- Accuracy is poor if loss is not compensated.
- Accuracy is much improved with loss compensation.
- Compensating loss with different zonal multiplier is slightly better than with a single multiplier.
- Compensating loss with a single multiplier in dc PF is an effective and perhaps the simplest way to improve the dc PF accuracy.

Bus Aggregation
A Different Approach

- Difference from modified Ward:
  - Aggregate buses rather than “eliminate” buses.
  - Match base case inter-zonal flows “accurately” (rather than retained transmission line flow as in modified Ward.)
- Aggregation algorithm applied to EI.
  - k-means ++
- Inter-zonal flow matching
- Test results for 6-bus system
- Tested on EI.

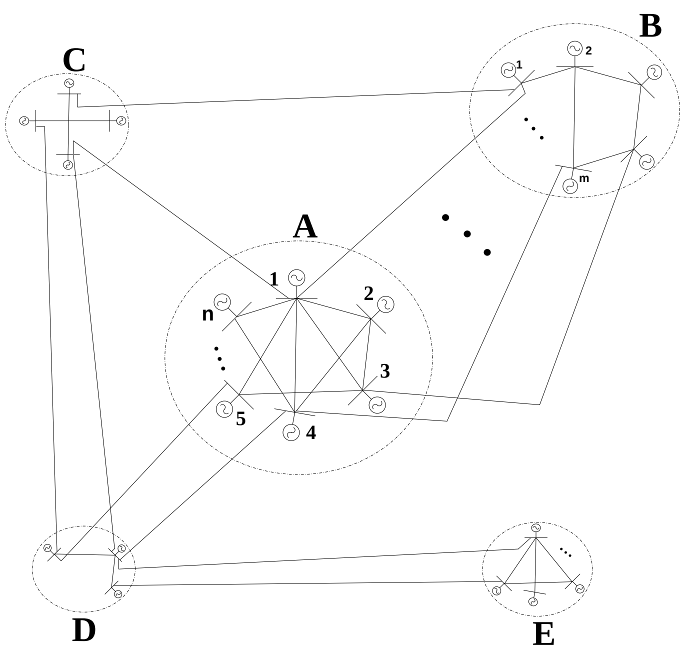
Bus Aggregation

- Basic idea:
  - Divide system into zones (aggregation.)
  - Each zone represented by a single bus
  - Generation & load aggregated to the single bus
  - Intra-zonal lines are neglected
  - Inter-zonal lines are aggregated
- Inter-zonal power flows reflect the bilateral transactions between two corresponding zones

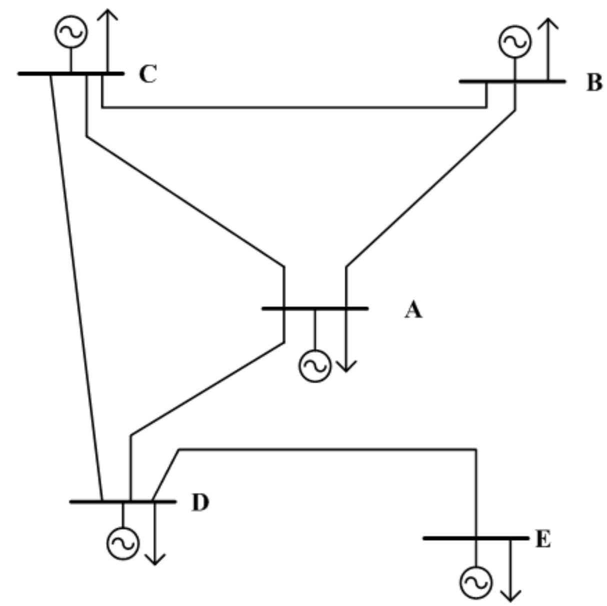


Topology of the Reduced Network

- Buses in the equivalent are connected by equivalent lines, if and only if, in the original system, there is at least one TL that directly connects the corresponding zones.



original

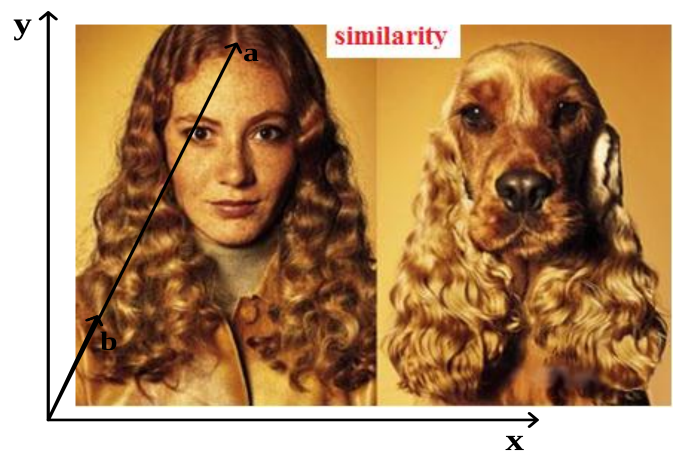


equivalent

Bus Clustering /Aggregation Algorithms

- Buses can be aggregated using any metric/philosophy
- To test the method, we aggregated buses according to the similarity of their injection effect on line flows (similar shift factors).
- Similarity Measures: cosine similarity vs. Euclidean distance
- Clustering algorithms: k-means, bisecting k-means, k-means++

Similarity of vectors in Euclidean n-space?

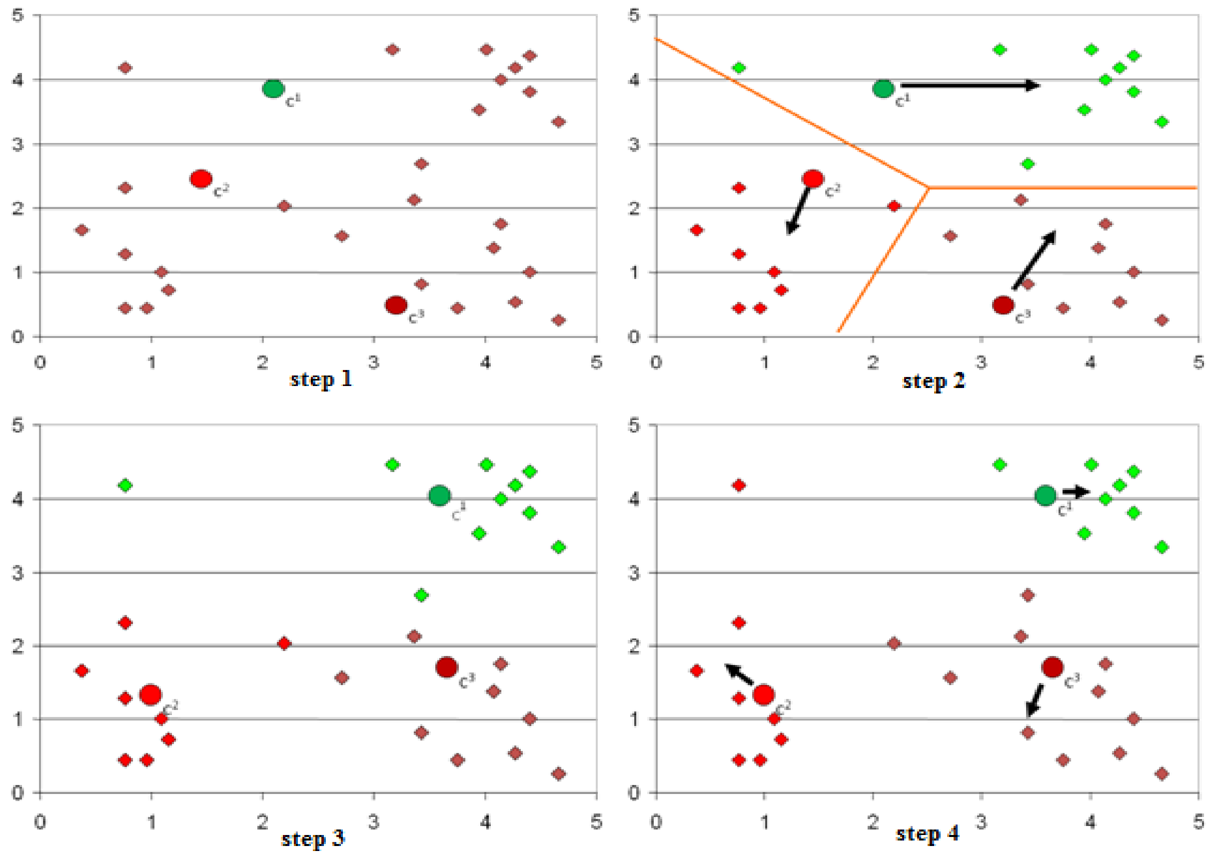


Power Transfer Distribution Factor (PTDF)

- PTDF matrix relates the power flows through TL's to the power injections at buses through a linear relationship.
- Similar line flow effect $\Leftrightarrow$ similar shift factors in PTDF matrix.
- Dimension & density of PTDF matrix.
- The EI data case we have contains approximately 60,000 buses and 80,000 branches. (EI$\Rightarrow$80,000 $\times$ 60,000$\approx$5 Billion entries)
- Computation of the full EI PTDF matrix uses parallel computing platform "Saguaro" (4560 processing cores) at ASU.

Clustering via K-means Algorithm

- The objective of the k-means algorithm works in such a way that within one cluster (*t*), the Euclidean distance between any bus and this cluster's center is smaller than the Euclidean distance between this bus and any of the other cluster's centers.
- This algorithm includes the following four steps:



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Bisecting K-means & K-means++

- NP hard problem
- K-means has problem when clusters are of different
  - Sizes
  - Densities
  - Non-globular shapes
- K-means has problem when the data contains outliers
- K-means may yield empty clusters for a large data set
- Result heavily depends on the initial seeding strategy
- To solve the problems of K-means, we tried
  - ✓ Bisecting K-means
  - ✓ K-means++

Bisecting K-means

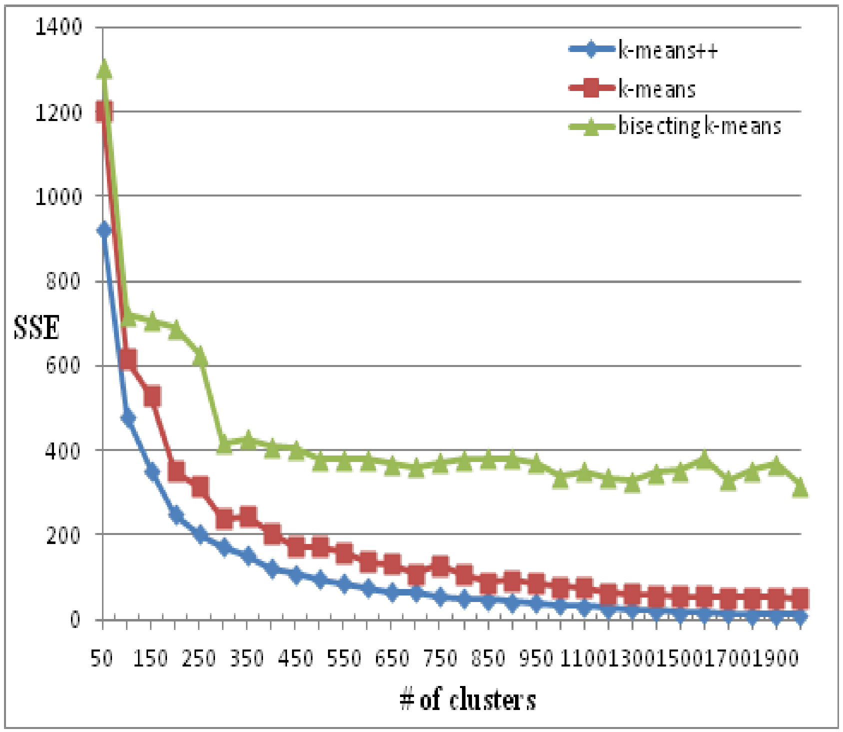
- Step 1: Search for $h_i^{(t)}$ and $h_j^{(t)}$ that belong to the same cluster S_t and correspond to terminals of one of the designated branches. If search fails (no vector is found), this process terminates; otherwise, go to step 2.
- Step 2: Split cluster S_t into two new clusters. Make $h_i^{(t)}$ and $h_j^{(t)}$ centroids of the two new clusters, which can be denoted as S_{hi} and S_{hj}. Evaluate each of the elements $h_k^{(t)}$ ($k \neq i$ and $k \neq j$) in S_t by calculating its Euclidean distance to $h_i^{(t)}$ and $h_j^{(t)}$. If $d(h_k^{(t)}, h_i^{(t)}) \leq d(h_k^{(t)}, h_j^{(t)})$, then $h_k^{(t)}$ should be assigned to cluster S_{hi}; otherwise, $h_k^{(t)}$ should be assigned to cluster S_{hj}.
- Step 3: Go back to step 1.

K-means ++

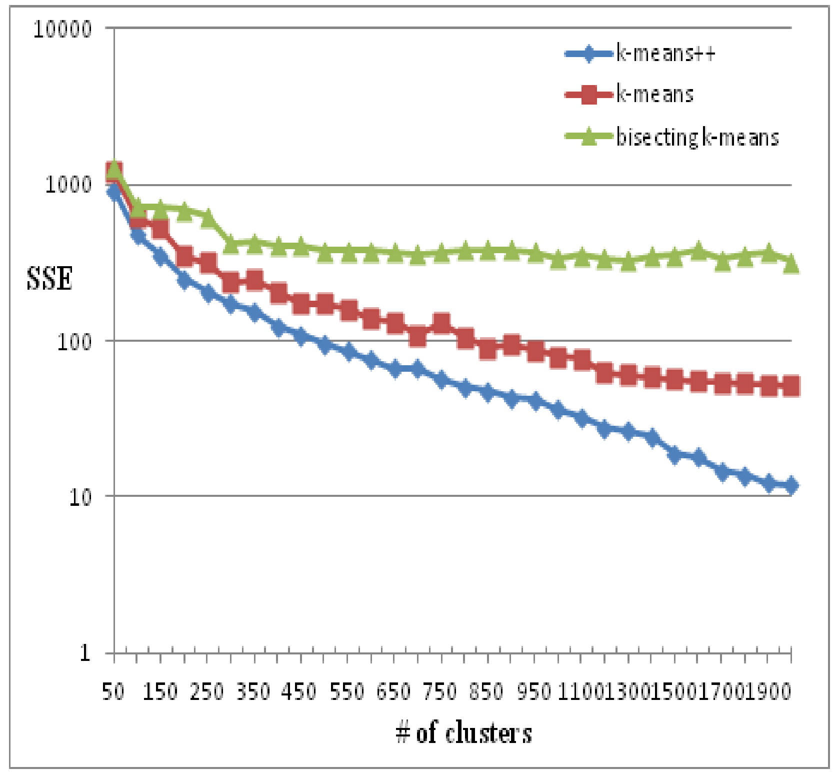
- Step 1: Initialize the list of clusters to a single cluster including all the vectors.
- Step 2: Choose a cluster from the list of clusters with the largest SSE. Remove this chosen cluster from the list of clusters.
- Step 3: Perform the *k*-means several times to bisect the selected cluster and form several bisections.
- Step 4: Select two clusters from the multiple bisection results with the smallest total SSE and add them to the list of clusters.
- Step 5: Check the number of clusters in the cluster list. If the desired number of clusters is reached, the clustering process terminates; otherwise, go back to step 2.

Clustering Results and Evaluation

- Most common measure is Sum of Squared Error (SSE)
  – For each point, the error is the distance to the nearest cluster centroid
  – To get SSE, we square these errors and sum them up



log scale



- Remark: Satisfactory results were obtained; K-means++ works the best.

Calculating Reduced Network Equivalent Reactances

- Calculate “effective” shift factors for reduced equivalent that match the inter-zonal flows of full network for the base case.
- Calculate set of branch reactance in reduced network which gives inter-zonal flows that are “closest” to those in the full matrix.

The Solution

$$(P_{inj})_R = \Pi_g P_{inj}$$

$$\Psi = \Pi_{flow} \Phi diag(P_{inj}) \Gamma$$

$$diag(P_{inj}) = \begin{bmatrix} P_{inj}^{(1)} & 0 & \cdots & 0 \\ 0 & P_{inj}^{(2)} & & \\ \vdots & & \ddots & \\ 0 & & & P_{inj}^{(N)} \end{bmatrix}$$

$$P_{flow}^{inter-zonal} = \begin{bmatrix} \sum_{j=1}^{N_R} \Psi^{(1,j)} \\ \sum_{j=1}^{N_R} \Psi^{(2,j)} \\ \vdots \\ \sum_{j=1}^{N_R} \Psi^{(L_R,j)} \end{bmatrix}$$

$$\Psi = \Phi_R diag\left[(P_{inj})_R\right]$$

$$\Psi = \Phi_R diag(\Pi_g P_{inj}) = \Pi_{flow} \Phi diag(P_{inj}) \Gamma$$

$$\Phi_R = \Pi_{flow} \Phi diag(P_{inj}) \Gamma \left\{ diag(\Pi_g P_{inj}) \right\}^{-1}$$

$$(\Phi_R C_R^T - I) diag(1/x_R) C_R = 0$$

$$(\Phi_R C_R^T - I) diag(1/x_R) c_i = 0, (i = 1, 2, \ldots, N_R)$$

$$(\Phi_R C_R^T - I) diag(1/x_R) c_i = 0$$
$$\leftrightarrow (\Phi_R C_R^T - I) diag(c_i)(1/x_R) = 0$$

$$\Lambda(1/x_R) = 0$$

$$\Lambda = \begin{bmatrix} (\Phi_R C_R^T - I) diag(c_1) \\ (\Phi_R C_R^T - I) diag(c_2) \\ \vdots \\ (\Phi_R C_R^T - I) diag(c_{N_R}) \end{bmatrix}$$

$$\theta_{ij}^* = \theta_i^* - \theta_j^*$$

$$P_{i->j} = \theta_{ij}^* / x_R^{(m)}$$

$$(1/x_R^{(m)}) = P_{i->j} / \theta_{ij}^*$$

$$\Lambda^*(1/x_R) = \begin{bmatrix} N_m \\ \Lambda \end{bmatrix} (1/x_R) = \begin{bmatrix} P_{i->j} / \theta_{ij}^* \\ 0 \end{bmatrix}$$

$$1/x_R = \left[(\Lambda^*)^T \Lambda^* \right]^{-1} (\Lambda^*)^T \begin{bmatrix} P_{i->j} / \theta_{ij}^* \\ 0 \end{bmatrix}$$

Finally arrives at

[B] D. Shi, D. Tylavsky, "An Improved Bus Aggregation Technique for Generating Network Equivalents," IEEE PES General Meeting, San Diego, CA, Jul 2012.

The Over-Determined Problem

- The linear over-determined problem and its solution:

$$\Lambda^*(1/x_R)=\begin{bmatrix} N_m \\ \Lambda \end{bmatrix}(1/x_R)=\begin{bmatrix} P_{i->j} / \theta_{ij}^* \\ 0 \end{bmatrix} \xrightarrow{\text{solution}} 1/x_R=\left[(\Lambda^*)^T\Lambda^*\right]^{-1}(\Lambda^*)^T\begin{bmatrix} P_{i->j} / \theta_{ij}^* \\ 0 \end{bmatrix}$$

  Number of rows in Λ is : nl•nb ≈6,000 •2,000≈12 million equations for a 2000 bus equivalent of the EI. Large computational burden.

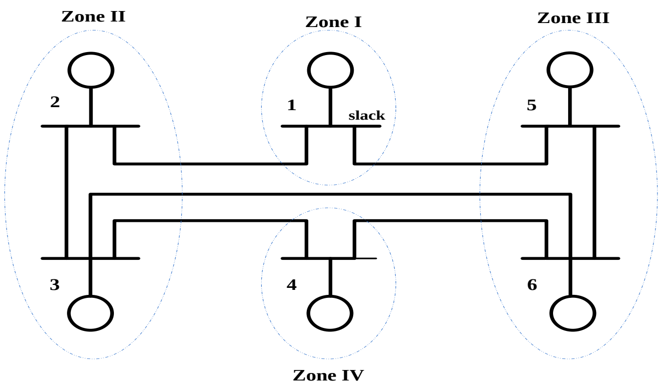
- To reduce the computational burden, the following features have been recognized:

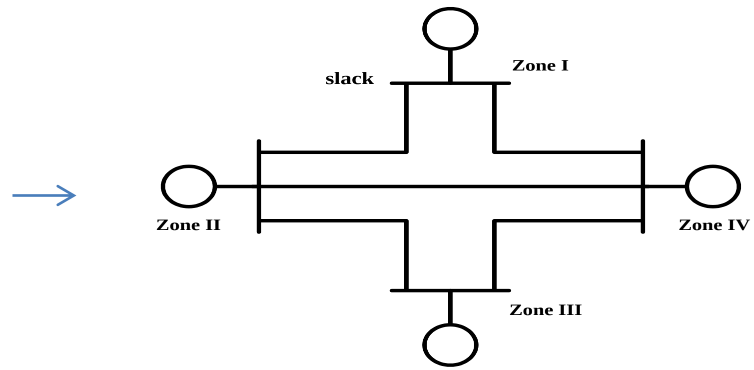
$$\Lambda=\begin{bmatrix} (\Phi_R C_R^T-I)diag(c_1) \\ (\Phi_R C_R^T-I)diag(c_2) \\ \vdots \\ (\Phi_R C_R^T-I)diag(c_{N_R}) \end{bmatrix}$$

  - Λ is a sparse matrix – very sparse
  - No. of non-zeros in each row of block '$(\Phi_R C_R^T - I)diag(c_i)$' equals to the no. of branches connected to bus *i*
  - In the equivalent, a bus is connected by 3~4 branches on average.
  - Structural property of the Λ matrix
  - Each block of '$(\Phi_R C_R^T - I)diag(c_i)$' contains 6000 equations, but only 4 are linearly independent.
  - Therefore, no need to calculate the full Λ matrix.

Numerical Example – Accuracy Validation 6-Bus System

- Apply approach to a small 6-bus test system.





Flow	Ward (MW)	Ref [5] (MW)	Proposed Method (MW)	Actual (MW)
$P_{I->II}$	-271.4	-244.8	-232.8	-232.8
$P_{I->IV}$	-128.6	-155.3	-167.2	-167.2
$P_{II->III}$	10.7	1.8	5.7	5.7
$P_{II->IV}$	17.9	53.5	61.5	61.5
$P_{III->IV}$	60.7	51.8	55.7	55.7
Avg. Error	**41.5%**	**20.1%**	**0%**	**NA**

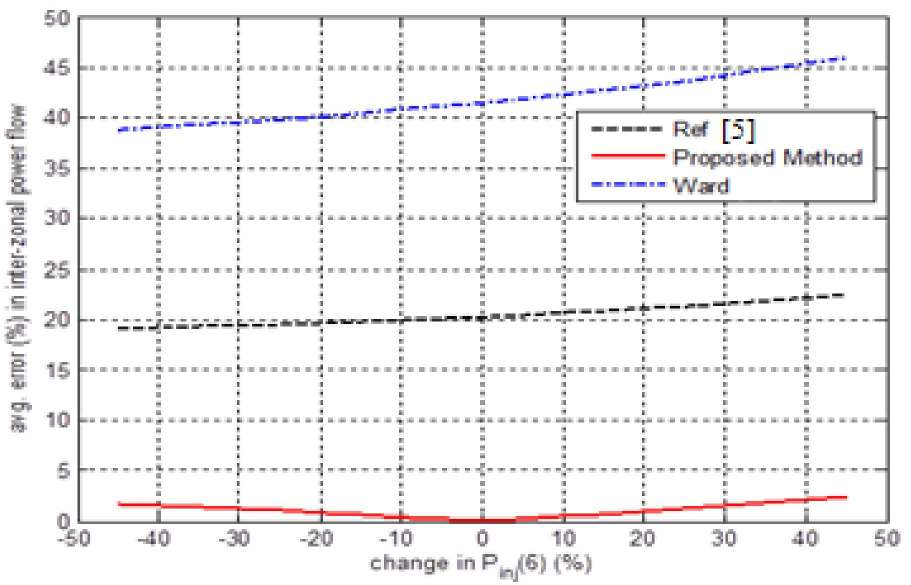
[5] H. Oh, "A New Network Reduction Methodology for Power System Planning Studies," *IEEE Trans. Power Syst.*, vol. 25, no. 2, May 2010.

Bus Aggregation Method Performance under Changed Operating Conditions

- The change in the system operating point is achieved by varying the power injection at bus #4 ($P_{inj}^{(4)}$).

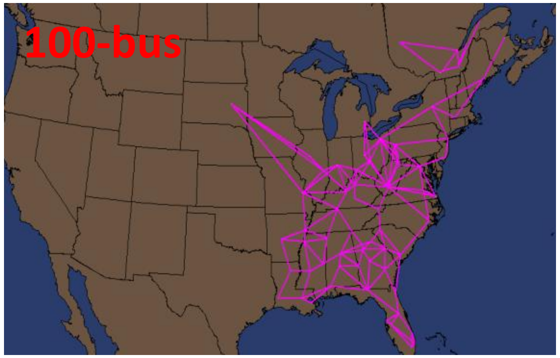
ERRORS IN THE INTER-ZONAL POWER FLOWS AS A FUNCTION OF CHANGE OF INJECTION AT BUS 6

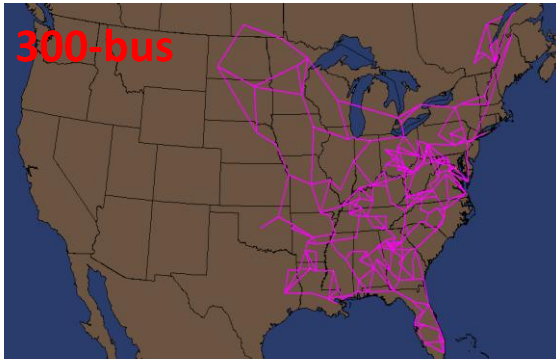
Change in $P_{inj}^{(6)}$ (%)	Average Error (%)		
	Ward	Ref [5]	Proposed Method
-45	38.87	19.10	1.74
-40	39.10	19.20	1.57
-30	39.60	19.42	1.21
-20	40.16	19.67	0.83
-10	40.79	19.96	0.43
0	41.50	20.28	0.01
10	42.30	20.65	0.45
20	43.20	21.07	0.95
30	44.22	21.55	1.49
40	45.39	22.10	2.07
45	46.04	22.41	2.39

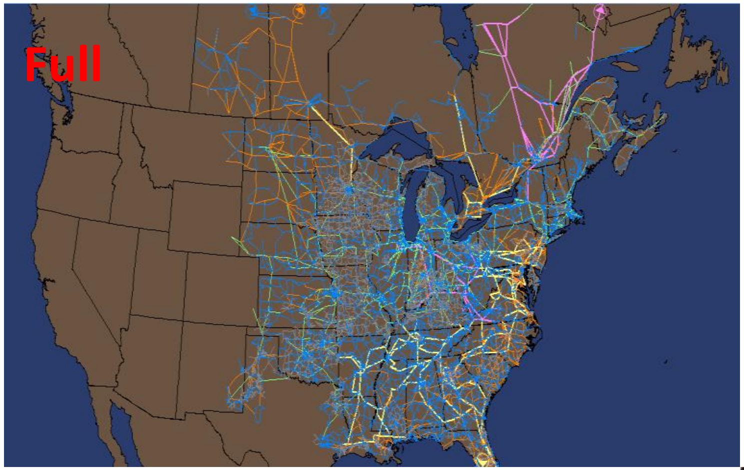


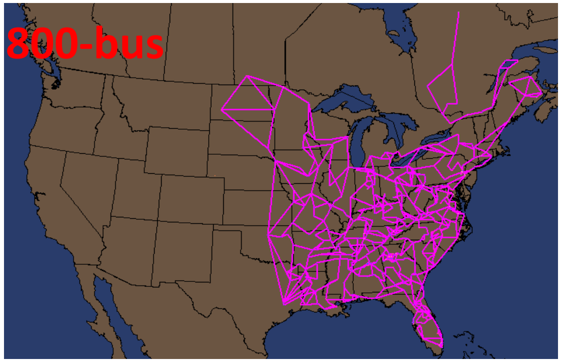
Average error in inter-zonal power flows as a function of the change in $P_{inj}^{(6)}$

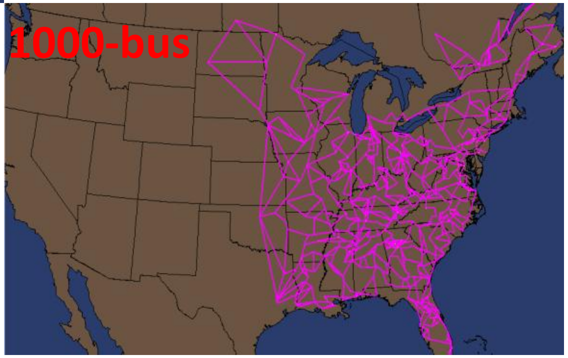
Application to EI





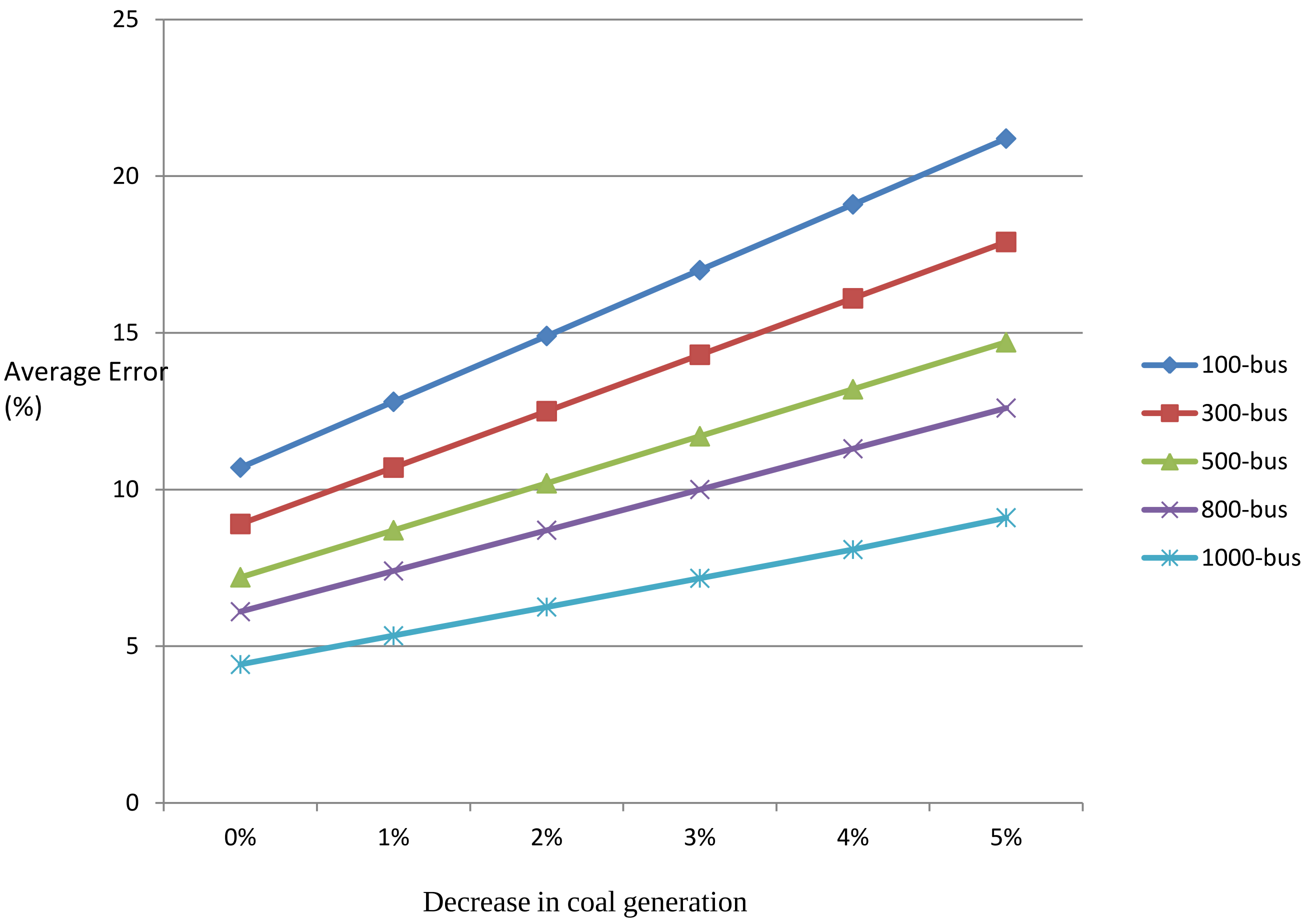






Execution Time Comparison

- For 1000-bus equivalent of the EI, the following accuracy is obtained.



Execution Time Comparison

- Both the proposed method and the method in [5] were coded and applied to the EI.
- To do a fair comparison, the same set of bus clustering results were used for both methods.
- For a 1000-bus equivalent of the EI, the following execution time were observed.

Execution Time for the Computation of..	Ref [5]	Proposed Method	Speedup Factor
Full PTDF Matrix	1.15 hr		-
Bus Clustering	4.5 hr		-
Reduced PTDF Matrix	4.4 hr	2.1 hr	**2.15**
Equivalent line reactance	80814 sec	18.09 sec	**447**

- Systems equivalents large than 1000 buses, the execution time of the bus clustering algorithm is too large even for our super computer:
  - Make the PTDF matrix sparse
  - Rewrite the k-means++ algorithm taking sparsity into account.
  - Use C++ instead of Matlab which alone is likely to give a speed up of 10 or greater.

[5] H. Oh, "A New Network Reduction Methodology for Power System Planning Studies," *IEEE Trans. Power Syst.*, vol. 25, no. 2, May 2010.

Conclusions

- Two network reduction methods were proposed and implemented for the continental U. S. power systems.
- Modified Ward Equivalents:
  - Equivalents were generated for the EI, WECC and ERCOT
  - The equivalents were validated using change-case PF solutions and dc OPF solutions.
  - With a "reasonable" size equivalent the modified Ward equivalent can yield accurate network solutions.

Conclusions

- A bus aggregation based network equivalent was developed and implemented in this work.
  - Three bus clustering algorithm were evaluated and the k-means++ approach showed the best result.
  - Method implemented and applied to 62,000 bus EI and an Eq generated
  - A large equivalent using this approach is expected to be suitable for accurately modeling inter-zonal flow in generation-investment planning studies.
  - The proposed bus aggregation based network reduction method is superior to existing bus-aggregation methods in:
    - base case performance (accuracy of inter-zonal power flow)
    - performance with changed operating conditions
    - computational efficiency

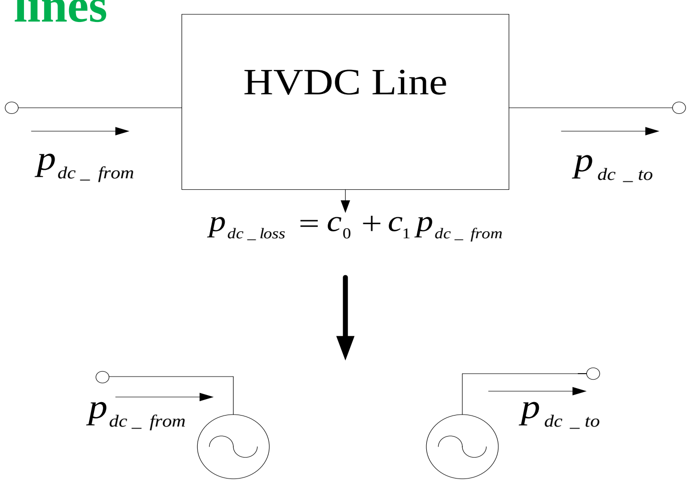
Questions?

Handling of Special Elements

-SVCs are converted to generators with no real power output

-Each HVDC line in the system is replaced by a pair of generators connected to two dummy buses.

-Allow optimal dispatch of HVDC lines



$$p_{dc\_loss} = c_0 + c_1 p_{dc\_from}$$

$$p_{loss} = p_{dc\_to} + p_{dc\_from}$$

$$p_{dc\_to} = (1 - c_1) p_{dc\_from} - c_0$$

-Islands: retain only large islands, remove small ones.

Slack Bus#	Slack Bus Area Name	Total Buses	Gen MW	Gen Mvar	Load MW	Load Mvar
180033	TE	1616	28010.4	3149.0	22568	5422.5
364003	TVA	59948	632976.7	119040.3	625196.3	171268.8
590002	ERCOT	6	622.3	249.6	400.1	0.4
590012	ERCOT	9	1802.1	370.2	1824.5	349.9
599956	WECC	3	606.2	199.9	500.0	0.0
652450	WAPA	2	5.1	36.1	85.1	28.1
652585	WAPA	4	111.4	57.8	2.0	0.0
659304	WAPA	2	0.0	-47.1	0.0	0.0
669732	MH	55	3560	1787.4	0.3	0.1

- 4. **Move** generators to the boundary buses which are closest to the generators.
  - The power network is a very complicated meshed network
  - A complicated algorithm was proposed to deal with the meshed network to move the generators.
  - The basic idea is dynamic programming

$$Dis_{ab} = \min_{k \in \{k_1 \dots k_n\}} \left(\sum_{k_i=1}^{m_i} \sqrt{r_{k_i}^2 + x_{k_i}^2} \right)$$

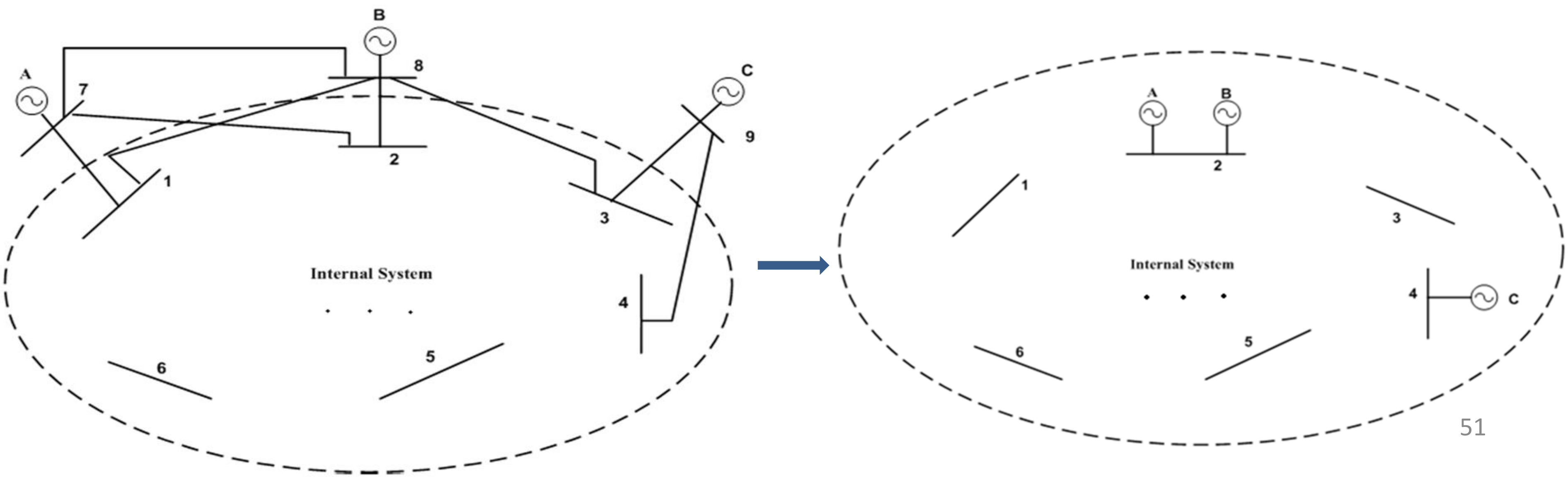
- Example

Line	$\sqrt{r^2 + x^2}$ (p.u.)
7-1	0.02
7-2	0.015
7-8	0.01
1-8	0.02
8-2	0.015
8-3	0.04
9-3	0.02
9-4	0.01

the distance between generator A and internal bus #1, #2 and #3 are calculated as:

$$Dis_{A1} = 0.02$$
$$Dis_{A2} = \min\{0.015, 0.025\} = 0.015$$
$$Dis_{A3} = \min\{0.05, 0.07\} = 0.05$$

generator A is electrically closest to bus #2, so that based on the proposed strategy, generator A is moved to bus #2.



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Model Evaluation-Dc OPF (EI-5222-bus equivalent)

```
The solution is optimal.

Converged in 35.89 seconds
Objective Function Value = 16244321.51 $/hr
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses            62013   Total Gen Capacity   872591.0         0.0 to 0.0
Generators        8152   On-line Capacity     717190.0         0.0 to 0.0
Committed Gens    5474   Generation (actual)  648157.7             0.0
Loads            30052   Load                 647872.3             0.0
  Fixed          30026     Fixed              647800.5             0.0
  Dispatchable      26     Dispatchable          71.8 of 525.5    -0.0
Shunts            3870   Shunt (inj)           -285.5              0.0
Branches         79222   Losses (I^2 * Z)         0.00             0.00
Transformers     79222   Branch Charging (inj)     -               0.0
Inter-ties        2477   Total Inter-tie Flow 253099.4             0.0
Areas              136

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   1.000 p.u. @ bus 100001      1.000 p.u. @ bus 100001
Voltage Angle    -130.08 deg   @ bus 129106    177.54 deg   @ bus 669755
Lambda P          -86.45 $/MWh @ bus 602025    265.42 $/MWh @ bus 200541
Lambda Q            0.00 $/MWh @ bus 100001      0.00 $/MWh @ bus 100001
```

```
Converged in 2.18 seconds
Objective Function Value = 16193495.27 $/hr
================================================================================
|     System Summary                                                           |
================================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses             5222   Total Gen Capacity   872732.0         0.0 to 0.0
Generators        8152   On-line Capacity     717331.0         0.0 to 0.0
Committed Gens    5474   Generation (actual)  648110.7             0.0
Loads             4124   Load                 648110.2             0.0
  Fixed           4098     Fixed              648086.2             0.0
  Dispatchable      26     Dispatchable          24.0 of 525.5    -0.0
Shunts               3   Shunt (inj)             -0.5              0.0
Branches         14225   Losses (I^2 * Z)         0.00             0.00
Transformers     14225   Branch Charging (inj)     -               0.0
Inter-ties        2027   Total Inter-tie Flow 197695.9             0.0
Areas              104

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   1.000 p.u. @ bus 100001      1.000 p.u. @ bus 100001
Voltage Angle     -85.35 deg   @ bus 126304    209.51 deg   @ bus 667016
Lambda P          -19.28 $/MWh @ bus 155073    109.56 $/MWh @ bus 130764
Lambda Q            0.00 $/MWh @ bus 100001      0.00 $/MWh @ bus 100001
```

COMPARISON BETWEEN THE DC OPF SOLUTIONS OF THE FULL AND EQUIVALENT EI MODELS

	Full EI Model	Equivalent EI Model
Convergence of the solution (Y/N)	Y	Y
Time for Convergence (sec)	35.89	2.18
Total Generation (MW)	648157.7	648110.7
Total Load (MW)	647872.3	648110.2[1]
Total Cost ($/Hour)	16,244,321.51	16,193,495.27
Average LMP ($/MWh)	50.6938	50.7067

The Flow Chart