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Development of Attribute Preserving Network Equivalents

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Overview

➢ Overall objective
  – To develop equivalent systems that preserve desired attributes of the original system

➢ Present focus
  – To create equivalents of interconnection level power grids that preserve line limits

➢ Special emphasis
  – To apply algorithm to a “backbone” type equivalent of large systems such as Eastern Interconnection (EI)
  – Key result is limits have been assigned to the EI equivalent provided by the Tylavsky group
For decades, power system network models have been equivalenced using the approach originally presented by J. B. Ward in 1949 AIEE paper “Equivalent Circuits for Power-Flow Studies”
Ward Equivalents

- Gaussian elimination on nodal admittance matrix
  - Also known as Kron reduction

- Admittance matrix updated by eliminating one bus at a time with partial factorization
  - More efficient than inverting admittance matrix
  - When bus $k$ between bus $i$ and bus $j$ is equivalenced

\[
y'_{ij} = y_{ij} - \frac{y_{ik}y_{kj}}{y_{kk}} = y_{ij} - \tilde{y}_{ij}
\]

- Equivalent line limits are missing
Summary of Accomplishments

- Developed algorithms for calculating equivalent line limits by matching total transfer capacity (TTC)
Previous Algorithms

- Used a sequential bus elimination approach, updating limits as buses were eliminated
  - Used Power Transfer Distribution Factors (PTDFs) and Total Transfer capability (TTCs) to calculate the limits
  - Even simple, unloaded systems could have no exact solution – required a range for the limits
  - Various algorithms were considered to determine the maximum upper/lower bounds for limits
    - Assignment problem approach (Hungarian algorithm) could result in wide limit ranges
    - Quadratic approach resulted in a smaller range, but could be quite expensive computationally
Previous Algorithm 118 Bus Case
Previous Algorithms 118 Bus Case
Reduced to 30 Buses

Black lines represent fully retained lines between buses from the original case. Green lines correspond to equivalent lines, now with limits.
Alternative, Top-down Approach

- Modified algorithm for large scale systems
  - Still based on TTC matching using PTDFs
- Creates equivalents from the system-level and proceeds downwards
  - Previous algorithm worked sequentially at the bus-level (bottom-up approach)
- Can handle loaded network
- Consists of two main parts
  - Creation of the equivalent without equivalent line limits
  - Assignment of equivalent line limits
Creation of Equivalent

Goal
- To create a backbone type equivalent without breaking up generation and possibly load injections

Method
- As with any equivalent, study/external buses must be selected a priori
- Explicitly retained external generators and loads are assigned to an internal bus; this defines a group

Criterion for grouping buses
- Substations have been used in this work
- Other criteria can also be used (application dependent)
Procedure for Creating Equivalent

- Select buses to be retained in the study system
- Assign each external bus to a study bus's group
- For each group
  - Move all generators, loads, and shunts from external buses to study buses; loads and/or generators can be split if desired
  - Combine any generators, load and/or shunts with similar types if desired
  - Move DC lines from external buses to study buses in both groups
  - Eliminate the external buses using Kron reduction
  - Discard equivalent lines above a desired impedance threshold
**Bus Grouping**

- Full system converted to equivalent system

Note: Only equivalent lines that are below the desired threshold are kept.
4 Bus Example

- Assume buses 1 and 3 are a group with bus 1 the only external bus
- When removing bus 1
  - Its generator is moved to bus 3
  - Bus 1 is eliminated using Kron reduction
  - Three equivalent lines are created between the other buses
Criteria for Moving Generators and Loads to Study Buses

During the Kron reduction process the equations solved are

\[
\begin{bmatrix}
I_e \\
I_s
\end{bmatrix} =
\begin{bmatrix}
Y_{ee} & Y_{es} \\
Y_{se} & Y_{ss}
\end{bmatrix}
\begin{bmatrix}
V_e \\
V_s
\end{bmatrix}
\]

\[
(I_s - Y_{se} Y_{ee}^{-1} I_e) = (Y_{ss} - Y_{se} Y_{ee}^{-1} Y_{es}) V_s
\]

How the generation and load should get moved is determined by the left-hand side vector

– This allocates the injection among the study buses
Criteria for Moving Generators and Loads to Study Buses

- One approach for discretely moving a generator to a study bus is to calculate how the generator's current injection would get allocated to the study buses, and then pick the one with the highest allocation
  - These allocations can be solved quite efficiently during the Ybus partial factorization
  - Would only need to be done for the largest generators
  - Allocations are usually quite localized, with significant values at just a handful of study buses
Eastern Interconnect Example

- Image shows percentage values of the UIUC generation to a 300+kV backbone equivalent

If generator is to be left whole, then it should be moved to the 53.3% substation (Sidney)
Assignment of Equivalent Line Limits

- Criteria for line limit preservation
  - Matching TTCs between pairs of buses for all equivalent lines with those of the same pairs in the original system (but source/sink may be distributed)

- TTC for transaction \( w \) between bus \( x \) and bus \( y \), \( P_w(x,y) \) can be calculated as

\[
P_w(x,y) = \min_{l_i \in L} \left\{ \frac{F_{l_i}}{\varphi_{l_i}^w(x,y)} \right\}
\]

- \( F_{l_i} \): limit of line \( l_i \) from the set of eliminated lines \( L \)
- \( \varphi_{l_i}^w(x,y) \): PTDF on line \( l_i \) for the same transaction \( w \)
Four Bus Example
(Exact solution case)

- PTDFs shown for bus 2-3
- When removing bus 1

\[
p_{w(2,3)} = \min \left\{ \frac{70}{0.32}, \frac{100}{0.26}, \frac{60}{0.06} \right\} = 217.0 \text{ MW (1-3 binding)}
\]

Likewise,

\[
p_{w(2,4)} = 171.7 \text{ MW (1-4 binding)}
\]

\[
p_{w(3,4)} = 145.0 \text{ MW (1-4 binding)}
\]
Assignment of Equivalent Line Limits

For the original system calculate the TTC for the transaction between buses $x$ and $y$, $P^w(x,y)$, only considering limits on lines to be eliminated
- Only the study buses are included in this calculation
- Distributed source and sink injections can be used

Using the equivalent system, calculate the PTDF on equivalent line $x$-$y$ for transaction bus $x$ to bus $y$, $\phi^w(x,y)$ $\phi^w(x,y)$

Assign limit of equivalent line $x$-$y$ as

$$\tilde{F}_{\tilde{l}(x,y)} = P^w(x,y) \times \phi^w(x,y)$$
Four Bus Example
(Exact solution case)

- PTDFs shown for bus 2-3
- Equivalent line limit 2-3 is

\[
\tilde{F}_{l(2,3)} = p^w(2,3) \times \tilde{\phi}_l^{w(2,3)} \\
= 217.0 \times 0.23 = 50.8 \text{ MVA}
\]

Likewise,

\[
\tilde{F}_{l(2,4)} = 171.7 \times 0.24 = 41.4 \text{ MVA} \\
\tilde{F}_{l(3,4)} = 145.0 \times 0.20 = 28.5 \text{ MVA}
\]
Four Bus Example
(Non-exact solution case)

Limit 1-4 reduced to 20 MVA from 60 MVA

<table>
<thead>
<tr>
<th></th>
<th>Top-down</th>
<th>Max/Hungarian</th>
<th>Quadratic Program</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper</td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>$\bar{F}_{l(2,3)}$</td>
<td>50.8</td>
<td>50.8</td>
<td>50.8</td>
</tr>
<tr>
<td>$\bar{F}_{l(2,4)}$</td>
<td>13.8</td>
<td>13.8</td>
<td>13.8</td>
</tr>
<tr>
<td>$\bar{F}_{l(3,4)}$</td>
<td>9.5</td>
<td>19.2</td>
<td>9.5</td>
</tr>
<tr>
<td>mismatch*</td>
<td>29.1%</td>
<td>18.2%</td>
<td>29.1%</td>
</tr>
</tbody>
</table>

*rms normalized TTC mismatch

New algorithm provides a lower bound on the limits
The next several slides provide examples of the application of the top-down algorithm applied to a 2012 EI model.

- Case originally had about 62,000 buses
- Simulations were done using SimAuto in PowerWorld and Matlab code
- Had to deal sometimes with case "quirks"
Negative Reactance Lines

- Testing indicates less likely to have exact solutions in networks with negative reactance (capacitive) lines
  - Negative reactances can cause PTDFs to have values above 100%
  - Negative reactances can occur on branches in series compensated lines, but the net line reactance is positive
  - EI case has about 1400 branches with X < 0; usually arise because of the modeling of three-winding transformers
Negative Reactance Lines, Four Bus Example

- Example shows previous four bus case with the reactance on the line from 2-3 changed from 0.08 to negative 0.08
  - PTDFs calculated for a transfer from bus 2 to bus 3
AEP and LGEE use 999 kV as nominal voltage for the star bus, others use 1 kV, some use the kV for one of the windings.

Image Source:
Working with the Tylavsky Group EI Model

- A desired outcome from the project is to provide a meaningful limits for the equivalent lines in the EI case being developed by Tylavsky group.

- A success of project is limits have been assigned, using the top-down approach, to the equivalent lines in this model.
  - Since an equivalent case was provided, there is no need to create the equivalent.
  - Prevent algorithm provides lower bound limits.
  - A next step is determine the impact of these limits.
Summary of EI Equivalent

<table>
<thead>
<tr>
<th></th>
<th>Original case</th>
<th>Equivalent case</th>
<th>( \frac{Eqv.}{Org.} \times 100 ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of buses</td>
<td>62,013</td>
<td>5,222</td>
<td>8.4</td>
</tr>
<tr>
<td>Number of branches</td>
<td>76,536</td>
<td>14,092</td>
<td>18.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(retained: 6,694, equivalent: 7,398)</td>
<td></td>
</tr>
<tr>
<td>Number of branches with negative reactance</td>
<td>1378 (average: -0.028 pu)</td>
<td>87 (average: -0.314 pu)</td>
<td>6.3</td>
</tr>
<tr>
<td>Number of branches with infinite limits</td>
<td>25,186</td>
<td>112</td>
<td>0.4</td>
</tr>
<tr>
<td>Number of overloaded original branches</td>
<td>0</td>
<td>61</td>
<td>n/a</td>
</tr>
<tr>
<td>Average % of original branch overloading</td>
<td>0</td>
<td>546</td>
<td>n/a</td>
</tr>
<tr>
<td>Generation</td>
<td>664,850 MW</td>
<td>664,691 MW</td>
<td>100.0</td>
</tr>
<tr>
<td>Load</td>
<td>647,898 MW</td>
<td>664,691 MW</td>
<td>102.6</td>
</tr>
</tbody>
</table>
Analysis of Equivalent (TTC matching)

- Table shows a comparison of study bus transactions between the original and the reduced system

| Transaction | Number of buses in between | $TTC_{org}$ (MW) | $TTC_{eq}$ (MW) | $\left| \frac{TTC_{org} - TTC_{eq}}{TTC_{org}} \right| \times 100$ (%) |
|-------------|---------------------------|------------------|-----------------|--------------------------------------------------------|
| A (500 kV - 500 kV) | 0                         | 1133.5           | 1099.5          | 3.0                                                    |
| B (765 kV – 765 kV) | 0                         | 4422.8           | 4385.1          | 0.9                                                    |
| C (765 kV – 345 kV) | 0                         | 4082.0           | 1891.4          | 53.7                                                   |
| D (500 kV – 345 kV) | 0                         | 2527.8           | 2524.4          | 0.1                                                    |
## Analysis of Equivalent (TTC matching)

- Transaction C also has negative reactance lines
- TTC mismatch is greatly reduced when there is no negative reactance lines involved in calculating the TTCs

| Transaction | Number of buses in between | $TTC_{org}$ (MW) | $TTC_{eq}$ (MW) | $\frac{|TTC_{org} - TTC_{eq}|}{TTC_{org}} \times 100$ (%) |
|-------------|-----------------------------|------------------|------------------|-------------------------------------------------|
| E (500 kV – 230 kV) | 4 | 826.3 | 829.0 | 0.3 |
| F (500 kV – 230 kV) | 4 | 2173.4 | 2061.4 | 5.2 |
| G (500 kV – 500 kV) | 4 | 9585.6 | 9594.0 | 0.1 |
| H (345 kV – 230 kV) | 3 | 471.7 | 464.2 | 1.6 |
Comments on Equivalent Line Limit Calculations

- PTDF threshold in TTC calculation
  - Lines with low PTDF would unlikely be the binding constraint, especially in backbone-type equivalent
  - With a 5% threshold, the number of lines to consider in the TTC calculation is reduced from total lines in case to an average of just dozens of lines

- Zero MVA line limits in the original case indicate no limit is enforced
  - Values replaced with either 9,999 MVA or they can be capped with a maximum power transfer value
    \[ \frac{V_1 V_2}{X} \sin \theta \]
Computational Aspects

- Problem formulation is straightforward compared to Quadratic program
- Computation is linear with respect to the number of equivalent lines
  - Applicable to large scale systems
  - In contrast, computation in bottom-up algorithms increased exponentially with respect to the number of first neighbor buses of eliminated buses/groups
- Two factors for control of computation vs accuracy
  - Impedance threshold to discard high impedance lines
  - PTDF threshold in calculating TTCs
Future Work

- Increase accuracy of the results
  - Develop algorithm to provide an upper bound on limits, in order to reduce large TTC mismatches
  - Dealing with lines with negative reactance

- Work with the Tyavsky group to determine impact of the limits on the solution results