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# Investigation of Advanced Stochastic Unit Commitment Solution for Optimal Management of Uncertainty

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# Overview

- Objective
- Proposed directions
  - Probabilistic constraints
  - Algorithms
- Challenges
- Conclusions

# What is “Advanced”?

- SCUC is a multi-stage and bi-level optimization problem
  - Level 1: Binary unit on/off decisions
  - Level 2: OPF/ED submodel for security
- Many variations on this structure to balance details and computational tractability
- Stochastic formulations have mainly been scenario-based with dc power flow

# Objective

- Investigation of a formulations for SCUC that will be
  - ✓ Flexible: able to include supply and demand side resources in a realistic way, integrate with other tools
  - ✓ Robust: provide optimal (or  $\epsilon$ -optimal) solutions
  - ✓ Scalable: applicable to reasonably-sized systems in practical computation time
- **Combination of formulation and algorithm implementation**

# Scenario-based optimization

- Most stochastic simulation is based on replication of deterministic models across scenarios
- Challenge of scenarios is probability selection
  - very low probability events are important
  - requires the weight of these events be reasonable to ensure their impact on the OF
- Rough representation of uncertainty with exact solutions versus detailed representation and approximate methods
- Constraints are pre-determined and must always hold

# Probabilistic Constraints

- We propose to investigate probabilistic (chance) constraint formulation
- Ease the requirement of perfect holding of constraints for low probability events
- Probabilistic constraints contain random variables and must be met with some (large) probability

# Probabilistic Constraints

- require that constraints that are random when the decision is made, should hold with high probability when the random variables are realized (Prékopa, 1995)
- These are often managed as penalty terms in the objective function

# Probabilistic Constraints vs Penalties

- Penalty is defined by the expectation that the constraint is unsatisfied
  - expectation is a long term average which is not reality for short-term planning
- Cost of violation of the constraint is frequently unknown
  - VOLL is usually used in power system applications
- Reliability of the system is not enforced specifically, but uses cost as a proxy
  - in wide-spread blackouts, for example, this may not suffice



# Comparing Traditional to Probabilistic

- Two-stage stochastic program with recourse:

$$\min (f(\mathbf{x}) + \mathbb{E} [G(\mathbf{x}, \boldsymbol{\xi})])$$

subject to

$$g_i(\mathbf{x}) \geq b_i$$

$$\mathbf{x} \geq 0$$

- where  $f(x)$  is the cost of the first stage problem and  $G(x, \xi)$  is the cost of the second stage (sub-problem)
- randomness in constraints are usually incorporated in the OF with penalty terms

# Chance-constrained formulation

- randomness remains in the constraints and is required to be met with some (high) probability

$$\min (f(\mathbf{x}) + \mathbb{E} [G(\mathbf{x}, \boldsymbol{\xi})])$$

subject to

$$\mathbb{P} (g_i(\mathbf{x}, \boldsymbol{\xi}) \geq \mathbf{b}_i) \geq p$$

$$\mathbf{x} \geq 0$$

- A similar two-stage dc formulation with chance-constraints has been proposed with wind uncertainty in (Wang et al, 2012)

# Challenges of Chance Constraints

- Most existing implementations are for linear problems
- Estimating the underlying distribution for the constraints
- Evaluating the distribution at each iteration is expensive -> scaling issues

# Solution Approaches

1. P-efficient points methods (Dencheva & Martinez, 2013)

side-step the expense of evaluating distributions through use of p-level efficient points

2. Regularization methods/bundle methods (Oliveira et al, 2011)

scale-up using inexact bundle methods that have reliable convergence if the distribution of r.v. is finite

3. Sample Average Approximation (Wang, 2012)

Using finite representation of probabilistic constraint/scenarios

4. Stochastic Dual Dynamic Programming (Philpott & Guan, 2008)



# p-efficient points

integration of multi-dimensional distribution of random variables with each iteration can be reduced with p-efficient points of the distribution.

- Optimality for log-concave distributions\*
- Provide upper and lower bound for arbitrary distributions
- Can use inexact information through Sample average approximation methods (SAA) or Bundle methods



# Sample Average Approximation

- SAA uses the empirical representative of distribution
- Essentially conduct monte carlo sampling of the underlying distribution (scenarios) of the second stage problem
- use as a representative of the true distribution when evaluating objective function
- convenient since distribution of many uncertainties (wind, for example) may be best represented empirically
- Wang (2012) estimate of bounds for both chance constraint and two-stage problem



# Bundle Methods

- Second stage optimization is completed using a subset of scenarios and “bundles” of function evaluations to represent the excluded scenarios
- Typically a “proximity” measure to bundle un-optimized scenarios (inexact bundle methods)
- Accelerates the evaluation of the second stage objective function (Oliveira et al, 2011)

# Stochastic Dual Dynamic Programming

- Decomposition method for multi-stage stochastic models
- Forward step: node sampling instead of solving all the nodes
- Backward step: solution of all the nodes of the recombining tree. It approximates the recourse function for the sampled values obtained in the forward step
- Stochastic convergence: lower bound (deterministic), upper bound (stochastic). (Philpott & Guan, 2008)
- Advantage: numerical complexity linear with number of scenarios



# Non-standard Resources

- Responsive demand is an important resource, but is typically incorporated as
  - lower cost load shedding (emergency resources)
  - demand elasticity (price-responsive)
- Responsive demand
  - exists at various time scales
  - is not deterministic
- We would like to incorporate these into the UC decision framework, but will increase dimensionality
- Flexible methods like SAA/inexact bundle, if scaled well, could incorporate DR



# Conclusions

- A number of promising approaches to explore
  - Smaller test cases will be evaluated this year, to filter the approach to most promising
    - Convergence
    - Scalability
  - Investigate methods for reduction of scenario sets and approximation of excluded scenarios

# Team

- Dr. Gabriela Martinez, Postdoctoral Associate
  - joining the project, Sept 1.
  - Probabilistic programming
  - Regularization methods
  - Experience in unit commitment

# References

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