Attribute Preserving Optimal Network Reductions

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Resources for the Future

Context

- Objective: Develop reduced-order network procedures that preserve voltage.
- IF you can preserve complex-value voltage phasors, then you can preserve most important quantities.
- Traditional network reductions (e.g., Ward & REI)
  - Use linearization somewhere in the process.
  - Theoretically exact only at a point—base case
- Objective: Develop network reduction
  - Preserve phasor voltages
  - Must preserve nonlinearities
- Achievement:
  - Network voltages theoretically exact along a (α) line--polynomial nonlinearities.
- Future
  - Preserve network voltages in a hyperplane—polynomial nonlinearities.
- Ultimate goal:
  - Application to the risk analysis problem when uncertainty is large.
  - (Important for voltage stability assessment.)

Outline

- Nonlinear Inverse functions: Use the Holomorphic Embedding Method (HEM) allowing nonlinear injections (Constant P/Q injections, ZIP loads)
- Network reduction via nonlinear inverse functions: radial and meshed systems.
- (Shruti Rao) Radial distribution system—Nonlinear two-bus phasor voltage-preserving model.
  - Convergence Issues: Power flow versus network equivalencing.
  - On the α line
  - Off the α line (estimating α)
- (Yujia Zhu) Meshed network—Generalizing the nonlinear phasor-voltage-preserving model reduction, arbitrary reduced-order model.
  - Derivation
  - Numerical experiments
  - Var limiting of external generators—theory and (prelim.) experiments.
  - Theory for moving along the $\alpha\Delta S$ line. (Shruti Rao and Yujia Zhu)
- Application of inverse functions to the risk analysis problem (future.)

Inverse Functions using HEM

- The power balance eq. (PBE) for a *PQ* bus can be written as:
$$\sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^*}{V_i^*}$$
- Holomorphically embedded as follows:
$$\sum_{k=1}^{N} Y_{ik} V_k(\alpha) = \frac{\alpha S_i^*}{V_i^*(\alpha^*)}$$
- With this embedding, α scales complex load, S.
- For holomorphic functions, $V(\alpha)$ is represented has Maclaurin series truncated N_T terms :

$$V(\alpha) = V[0] + V[1]\alpha + V[2]\alpha^2 + \ldots + V[N_T]\alpha^{N_T}$$

Inverse Functions using HEM

- Express inverse of $V_i(\alpha)$ series on the RHS as an inverse series $W(\alpha)$ where $W_i(\alpha) = \dfrac{1}{V_i(\alpha)}$

- Thus the PBE is represented as:

$$\sum_{k=1}^{N} Y_{ik}(V_k[0] + V_k[1]\alpha + V_k[2]\alpha^2 + \ldots + V_k[N_T]\alpha^{N_T}) =$$

$$\alpha S_i^*(W_i^*[0] + W_i^*[1]\alpha + W_i^*[2]\alpha^2 + \ldots + W_i^*[N_T]\alpha^{N_T})$$

- The solution at α=0 (germ) : $\sum_{k=1}^{N} Y_{ik} V_k[0] = 0$

- Subsequent series terms obtained through a recurrence relation obtained by equating like powers of α on both sides.

$$\sum_{k=1}^{N} Y_{ik} V_k[n] = S_i^* W_i^*[n-1]$$

Inverse Functions using HEM

- Similarly the equations for PV buses can be embedded as follows:

$$\sum_{k=1}^{N} Y_{ik} V_k(\alpha) = \frac{\alpha P_i - jQ_i(\alpha)}{V_i^*(\alpha^*)} \qquad V_i(\alpha) * V_i^*(\alpha^*) = \left|V_i^{sp}\right|^2$$

where P_i is the known power injected into the bus and V_i^{sp} is the specified voltage for the PV bus.

- The embedded equation for the slack bus is given by:

$$V_{slack}(\alpha) = V_{slack}^{sp}$$

- Combining the slack, PQ and PV bus equations, the PBE's of a power system can be solved recursively to obtain the terms of the voltage power series.

Inverse Functions using HEM

Analytic Continuation via Padé Approximants

- Challenge: The voltage power series may not always converge.
- Padé approximants are used to obtain a converged solution, if it exists.
- Stahl's Padé convergence theory- For an analytic function with finite singularities, the sequence of near-diagonal Padé approximant converges to the function... [1]
- Padé approximants are rational approximants to the given power series given by:

$$V(\alpha S) = V[0] + V[1]\alpha + V[2]\alpha^2 + \ldots V[L+M]\alpha^{L+M} + O(\alpha^{L+M+1})$$

$$= \frac{a_0 + a_1\alpha + a_2\alpha^2 + \ldots + a_L\alpha^L}{b_0 + b_1\alpha + b_2\alpha^2 + \ldots + b_M\alpha^M} = \frac{a_L(\alpha)}{b_M(\alpha)}$$

[1] H. Stahl, "On the Convergence of Generalized Padé Approximants," Constructive Approximation, 1989, vol. 5, pp. 221–240.

Padé approximants
Power Flow Convergence Issues

- **Power flow convergence** check diagonal approximants at all/sentinel nodes at α=1:

$$V(\alpha)_{[M/M]} = \frac{a_0 + a_1\alpha + a_2\alpha^2 + \ldots + a_M\alpha^M}{b_0 + b_1\alpha + b_2\alpha^2 + \ldots + b_M\alpha^M} = \frac{a_M(\alpha)}{b_M(\alpha)}$$

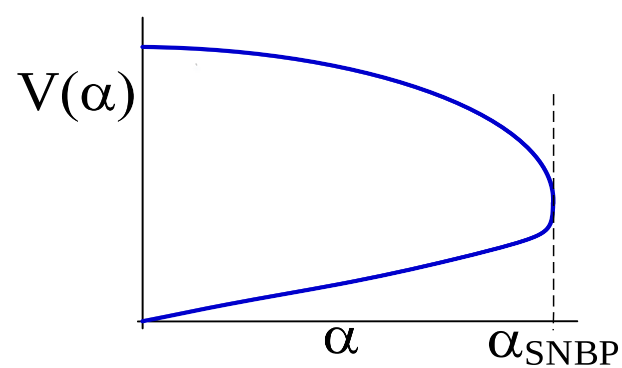
$$\left\| \left|\frac{a_{M+1}(\alpha)}{b_{M+1}(\alpha)}\right| - \left|\frac{a_M(\alpha)}{b_M(\alpha)}\right| \right\|_{\alpha=1} = \left\| \left|V(\alpha)_{[M+1/M+1]}\right| - \left|V(\alpha)_{[M/M]}\right| \right\|_{\alpha=1} < 10^{-4}$$

  – We then check for bus-power mismatches < 0.1 MW (transmission).

- **Convergence check for generating equivalents**.
  – Diagonal approximants at all/sentinel nodes at α=SNBP.

$$\left\| \left|\frac{a_{M+1}(\alpha)}{b_{M+1}(\alpha)}\right| - \left|\frac{a_M(\alpha)}{b_M(\alpha)}\right| \right\|_{\alpha=SNBP} < 10^{-4}$$

  – Must predict the SNBP.

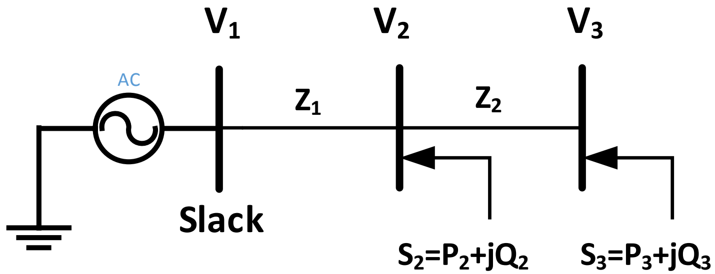


Padé approximants
Equivalents-Generation Convergence Issues

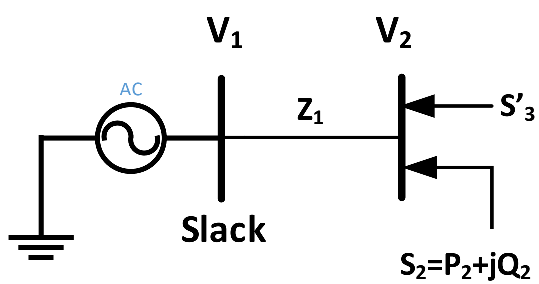
- (Meshed Systems) Temporary work around.
  - Use 120 terms in the series.
  - Check condition number of Padé matrix.
  - Check Padé matrix equation solution *Ax=b* accuracy.
- (Radial Systems) Economical convergence check for generating equivalents.
  - Metrics for checking for SNBP convergence.
  1. SNBP change. $\left|SNBP_{2M+K} - SNBP_{2M}\right| < \varepsilon$
  2. Power mismatch < 0.1 MW
  3. Padé approximant change. $\left\| \left| \frac{a_{M+K/2}(\alpha)}{b_{M+K/2}(\alpha)} \right| - \left| \frac{a_M(\alpha)}{b_M(\alpha)} \right| \right\|_{\alpha = SNBP_{2M+K}} < 10^{-4}$
  - SNBP may be estimated by:
    - 1) α-line binary search (experimenting)-sentinel node selection?
    - 2) roots method—finding smallest real root of high order numerator/denominator polynomial.
  - Selected: #1 (SNBP convergence) and #2 (P-mismatch)
    - Used both line search and roots method with similar results on radial systems.

Ward Radial Network Reduction

- Consider a three-bus network as shown below. SNBP=3.8×Base_Load



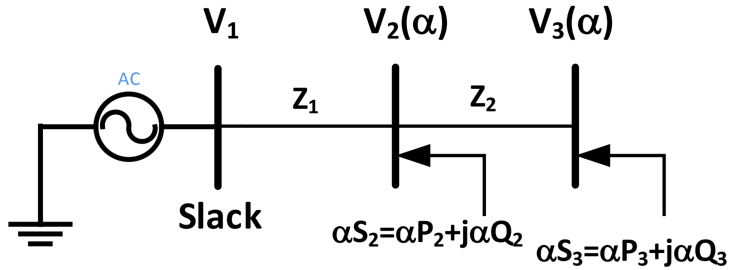
- Ward reduction: Convert P_3+jQ_3 load to current injection: $I^*_3=(P_3+jQ_3)/V_3$
- Eliminate bus 3 using Ward reduction method—move I_3 bus 2.
- Convert I_3 to equivalent $S'_3= I^*_3\ V_2$ load at buses 2.



- Compare with using HEM approach.

Analytic Reduction Method (ARM) Radial Network

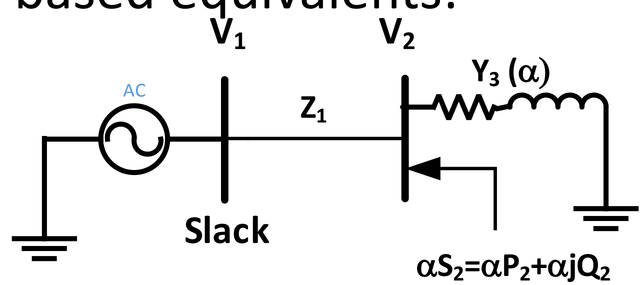
Consider a three-bus ***HEM-solved*** PF problem, shown as the network below.



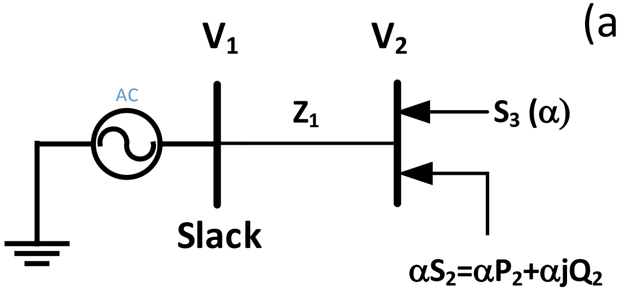
Bus number	P (pu)	Q (pu)	V (pu)
1	0.0	0.0	1.0
2	0.05	0.02	0.991
3	0.23	0.08	0.959

From Bus	To bus	R (pu)	X (pu)
1	2	0.05	0.13
2	3	0.08	0.15

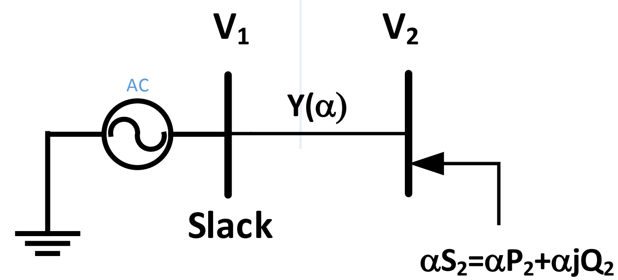
- Three HEM-based equivalents:



(a) Shunt $Y(\alpha)$



(b) Shunt $S(\alpha)$



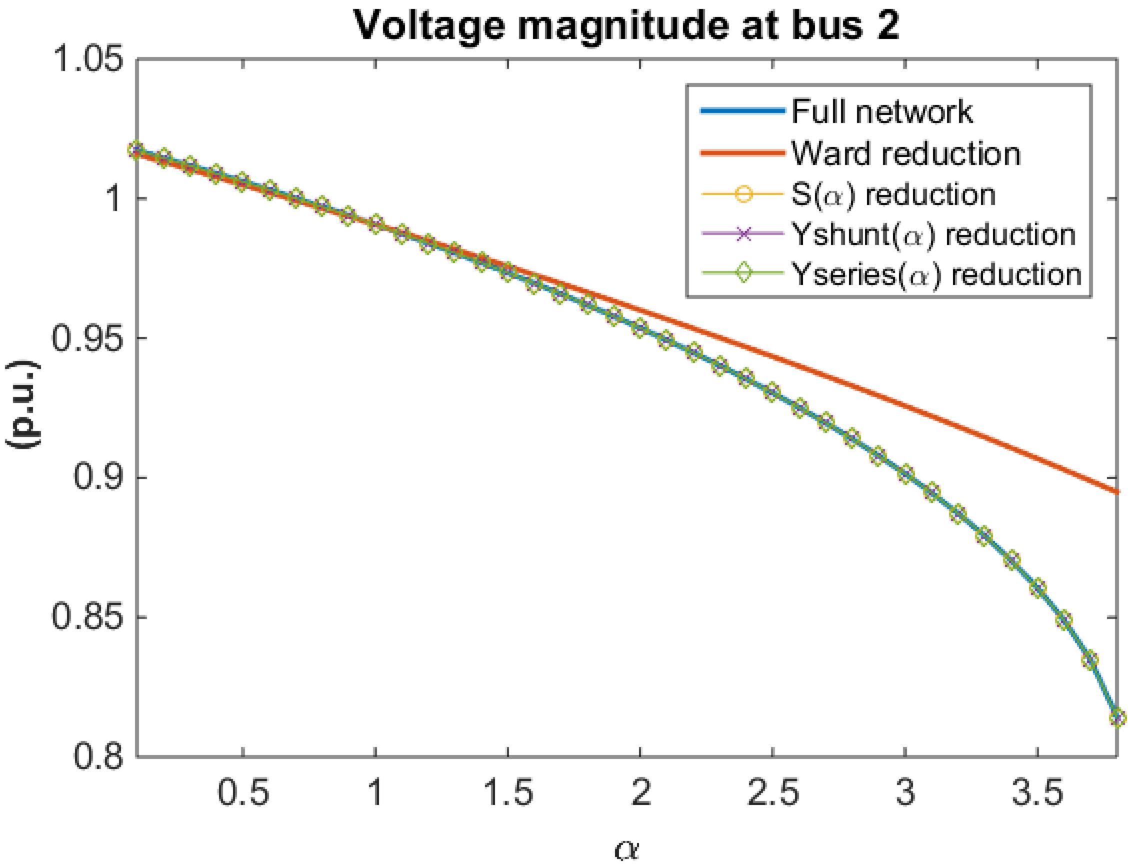
(c) Series $Y(\alpha)$

$$S_3(\alpha) = V_2(\alpha)\frac{(V_2^*(\alpha) - V_3^*(\alpha))}{Z_2^*}$$

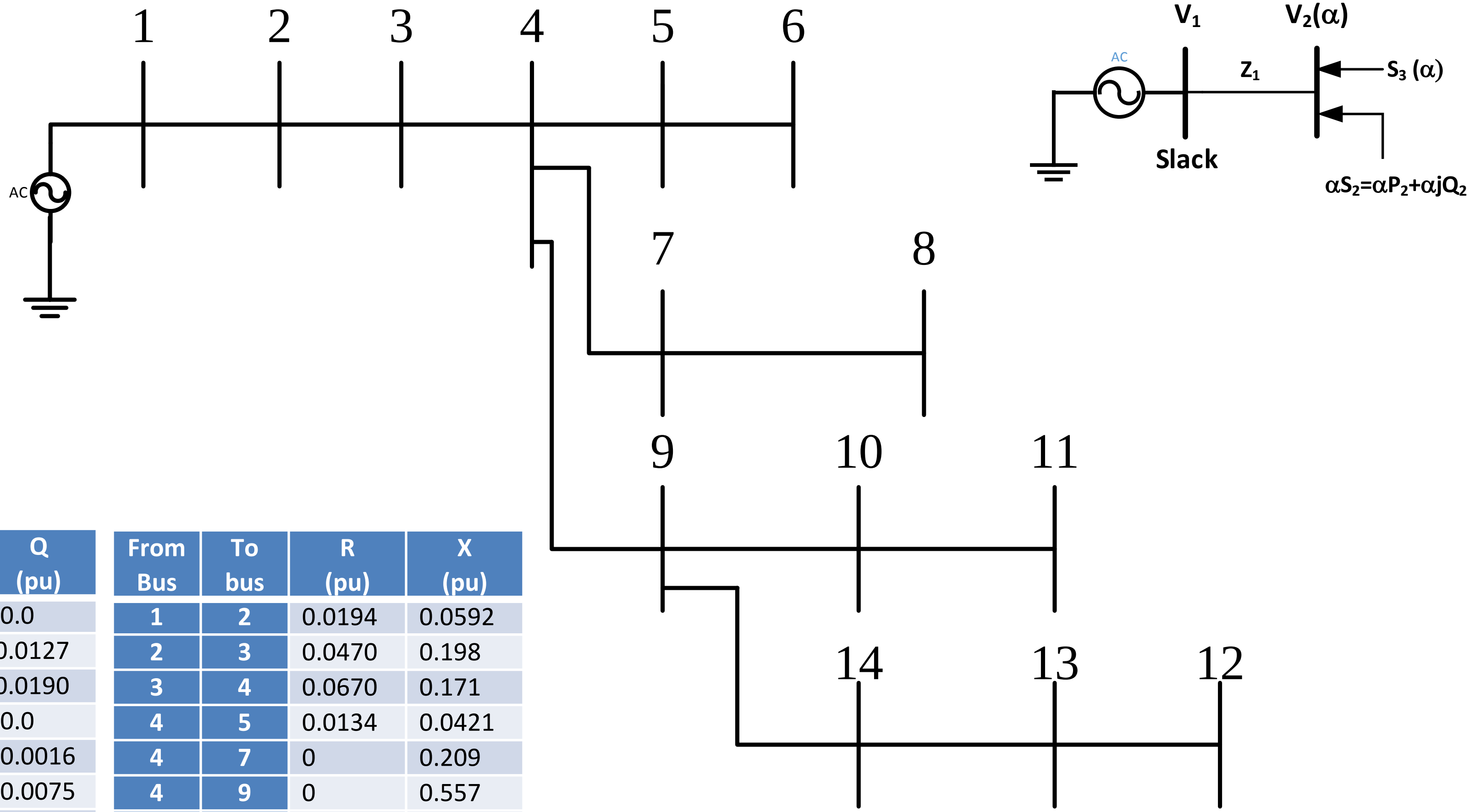
Accuracy Test: Scale loads uniformly to voltage collapse.

Inverse Function/HEM
3-Bus Radial Network Reduction

- Static voltage collapse point:
  - Unreduced Network: SNBP=3.8×Base load
  - Ward Reduction: SNBP=7.5
  - Inverse Function Approach: SNBP=3.8
- Bus 2 voltage plot (S_2=0.)



14-Bus Radial to 2-Bus Radial Network Reduction--On the α Line

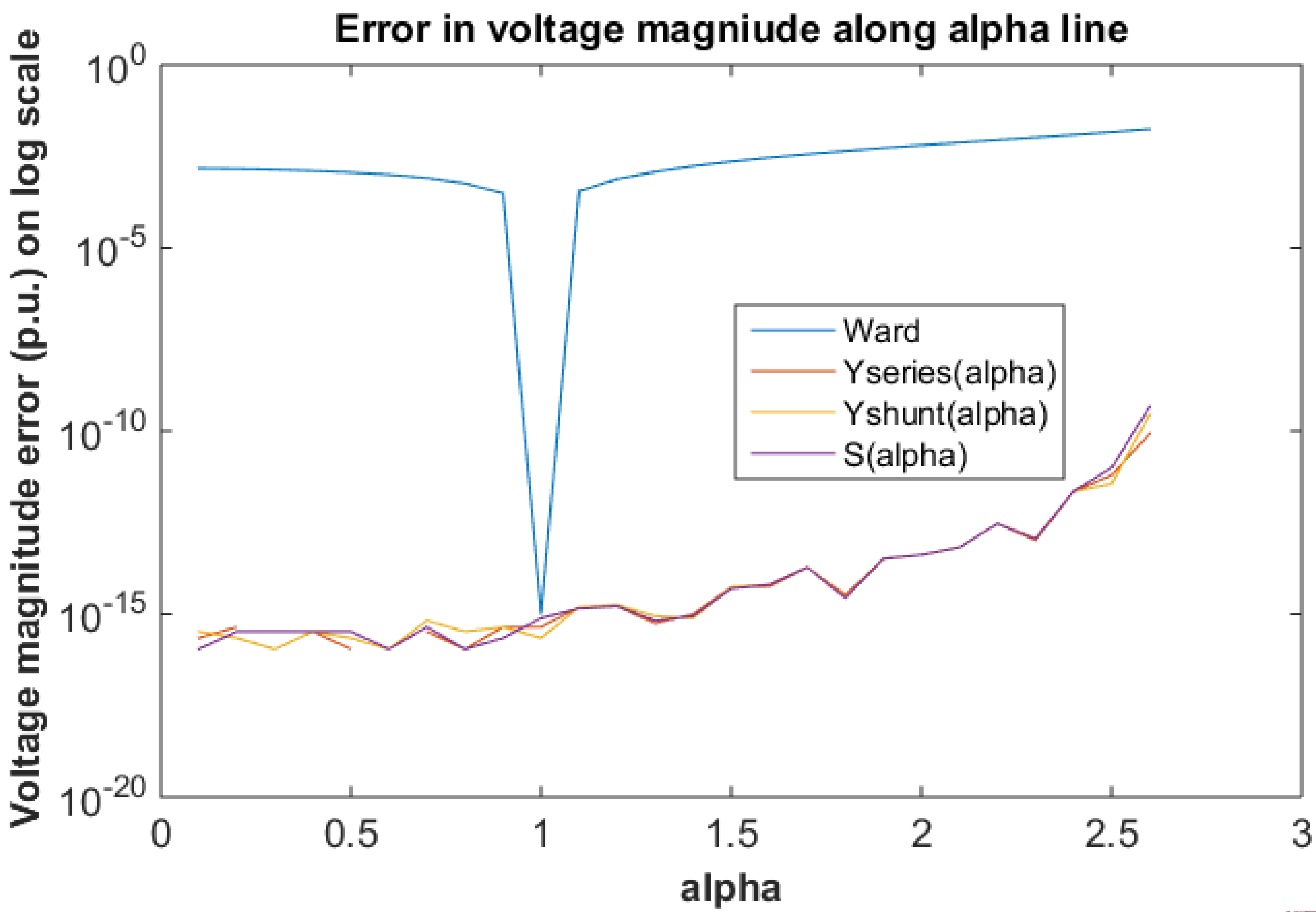


Bus number	P (pu)	Q (pu)
1	0.0	0.0
2	0.0217	0.0127
3	0.0942	0.0190
4	0.0478	0.0
5	0.0076	0.0016
6	0.0112	0.0075
7	0.0	0.0
8	0.0	0.0
9	0.0295	0.0166
10	0.009	0.0058
11	0.0035	0.0018
12	0.0061	0.0016
13	0.0135	0.0058
14	0.0149	0.005
Total	**0.259**	**0.0774**

From Bus	To bus	R (pu)	X (pu)
1	2	0.0194	0.0592
2	3	0.0470	0.198
3	4	0.0670	0.171
4	5	0.0134	0.0421
4	7	0	0.209
4	9	0	0.557
5	6	0	0.252
7	8	0	0.176
9	10	0.0318	0.0845
9	14	0.127	0.270
10	11	0.0820	0.192
12	13	0.221	0.200
13	14	0.171	0.348

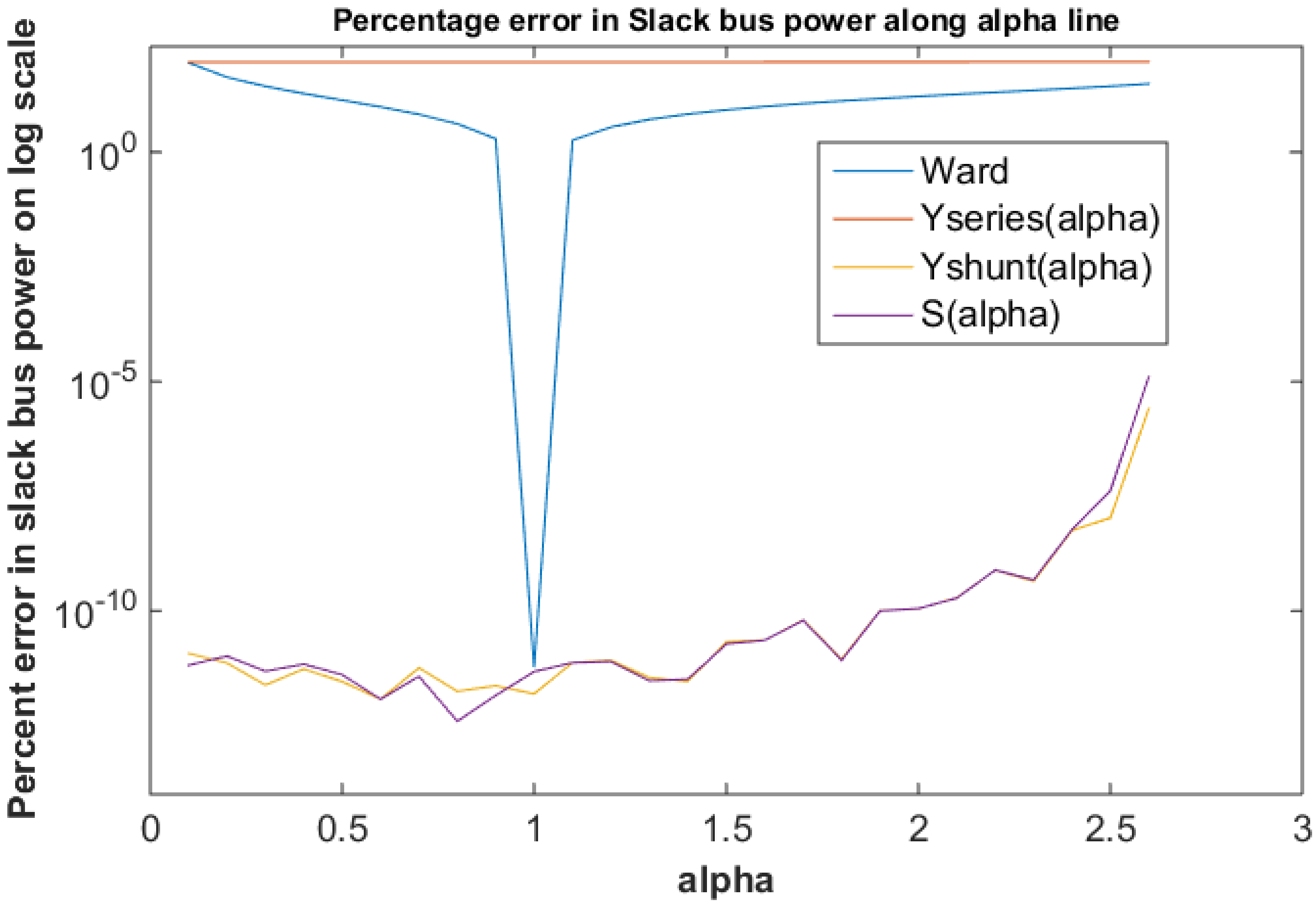
14-Bus Radial to 2-Bus Radial Network Reduction—On the α line

- Voltage mag error on the α line.
- SNBP=2.6α



14-Bus Radial to 2-Bus Radial Network Reduction—On the α line

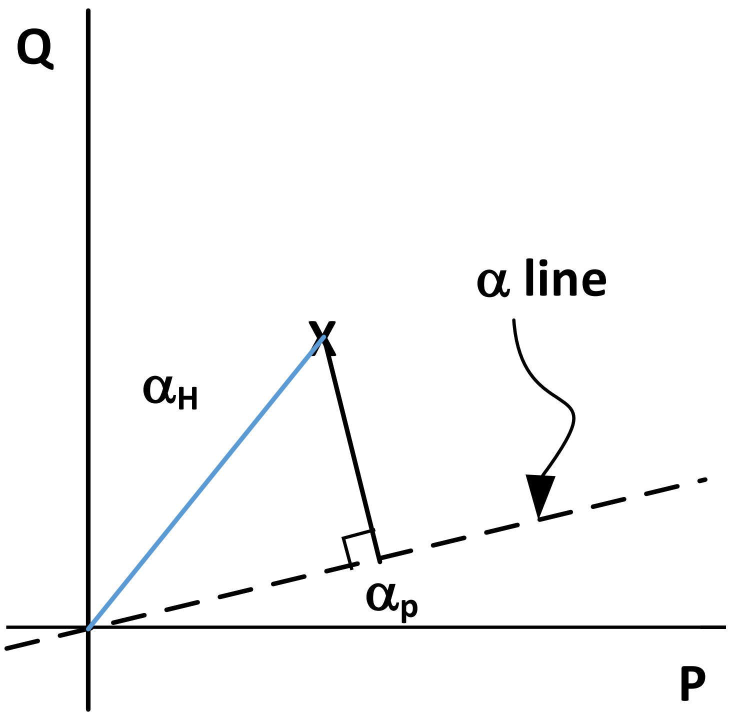
- Percent error in slack bus power on the α line.
- SNBP=2.6α
- HEM reduction approach superior to Ward reduction on the α line.



Getting off the α line

Estimating Equivalent α Network Reduction—Off the α line

- HEM: How to estimate equivalent α?—Five methods:
  - α_P—Length of orthogonal projection onto the α line.
  - α_H—|Sum (P+jQ)$_{new}$|/ |Sum (P+jQ)$_{old}$|
  - Etc.



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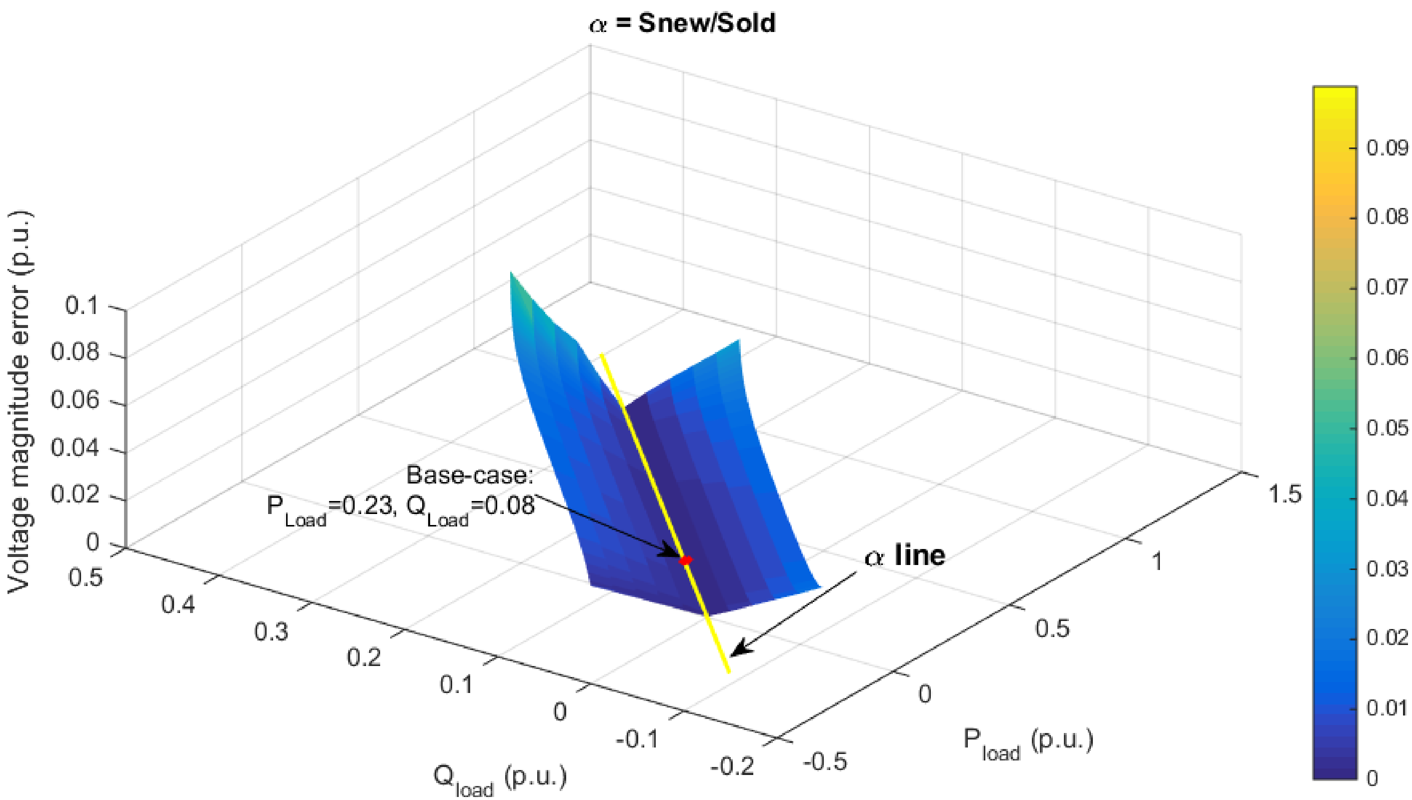
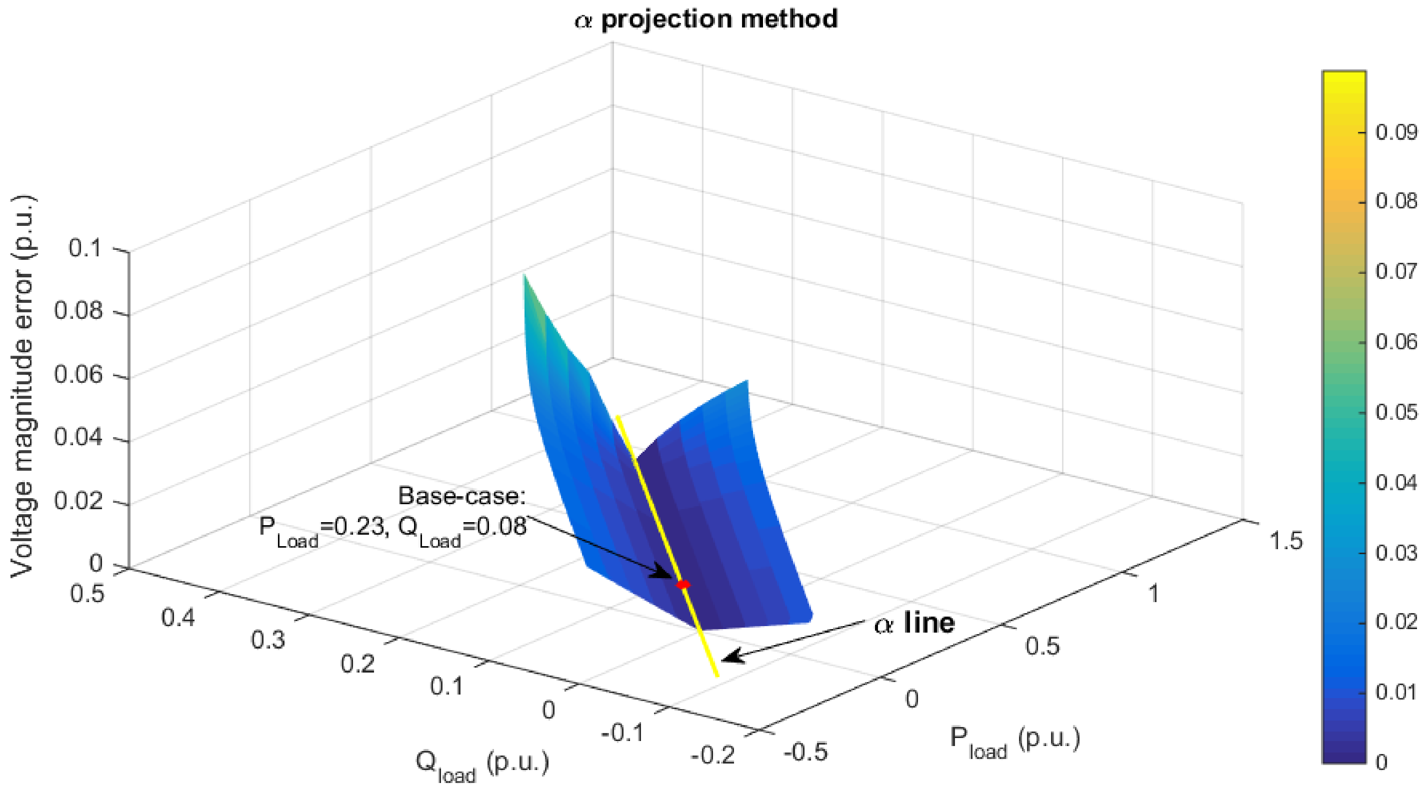
3-Bus Radial to 2-Bus Radial Network Reduction—Off the α line

- HEM: Reduced-order networks exact along the α line.
- α_H-$|S_{new}|/|S_{old}|$ Method: Bus 2 voltage magnitude error.
- Load scaled by factor of 4.
- Load variation +/-50% of base case load.



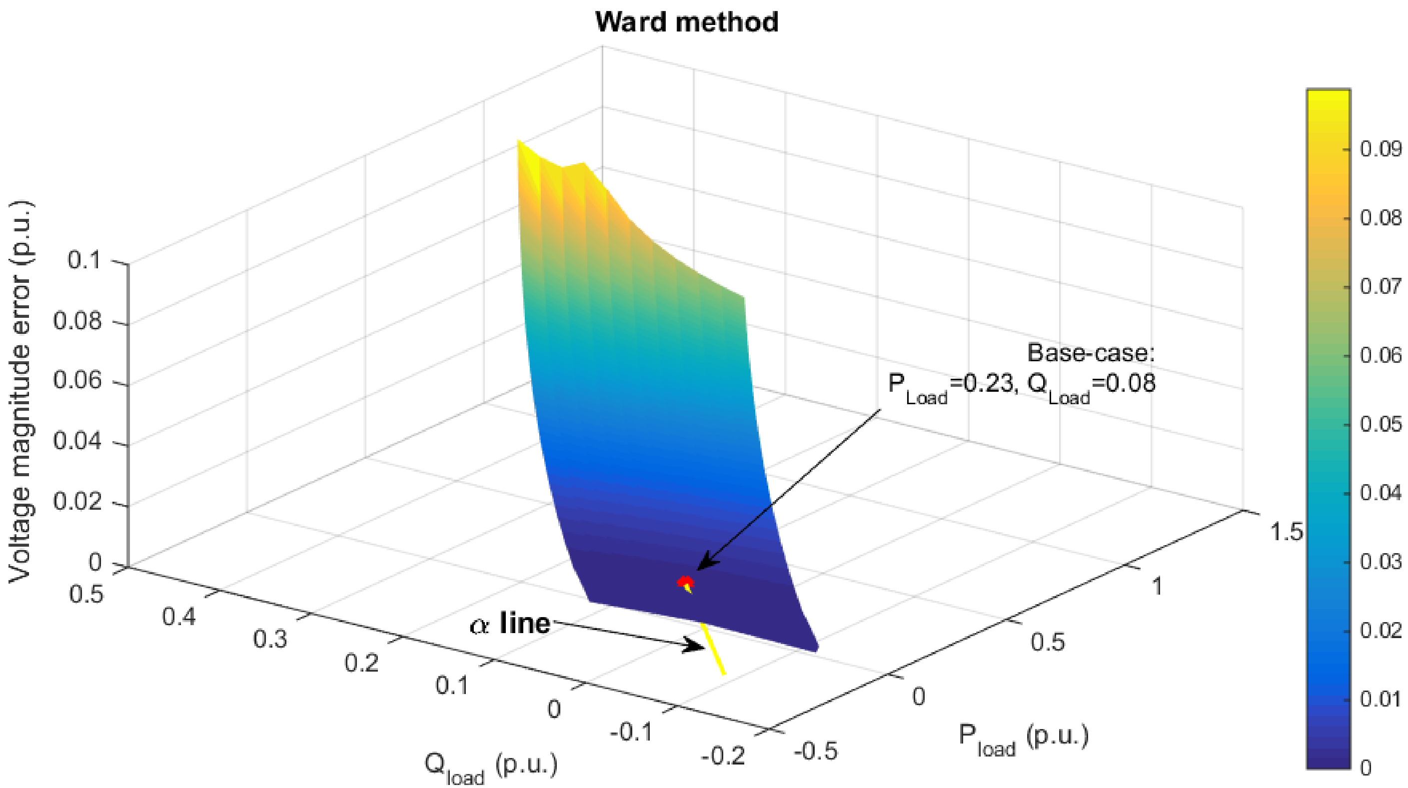
3-Bus Radial to 2-Bus Radial Network Reduction—Off the α line

- α_P-Projection Method: Bus 2 voltage magnitude error.
- Load scaled by factor of 4.
- Load variation +/-50% of base case load.



3-Bus Radial to 2-Bus Radial Network Reduction—Off the α line

- Ward Method: Bus 2 voltage magnitude error—exact at one point.
- Ward: Add change in load to boundary bus injection.

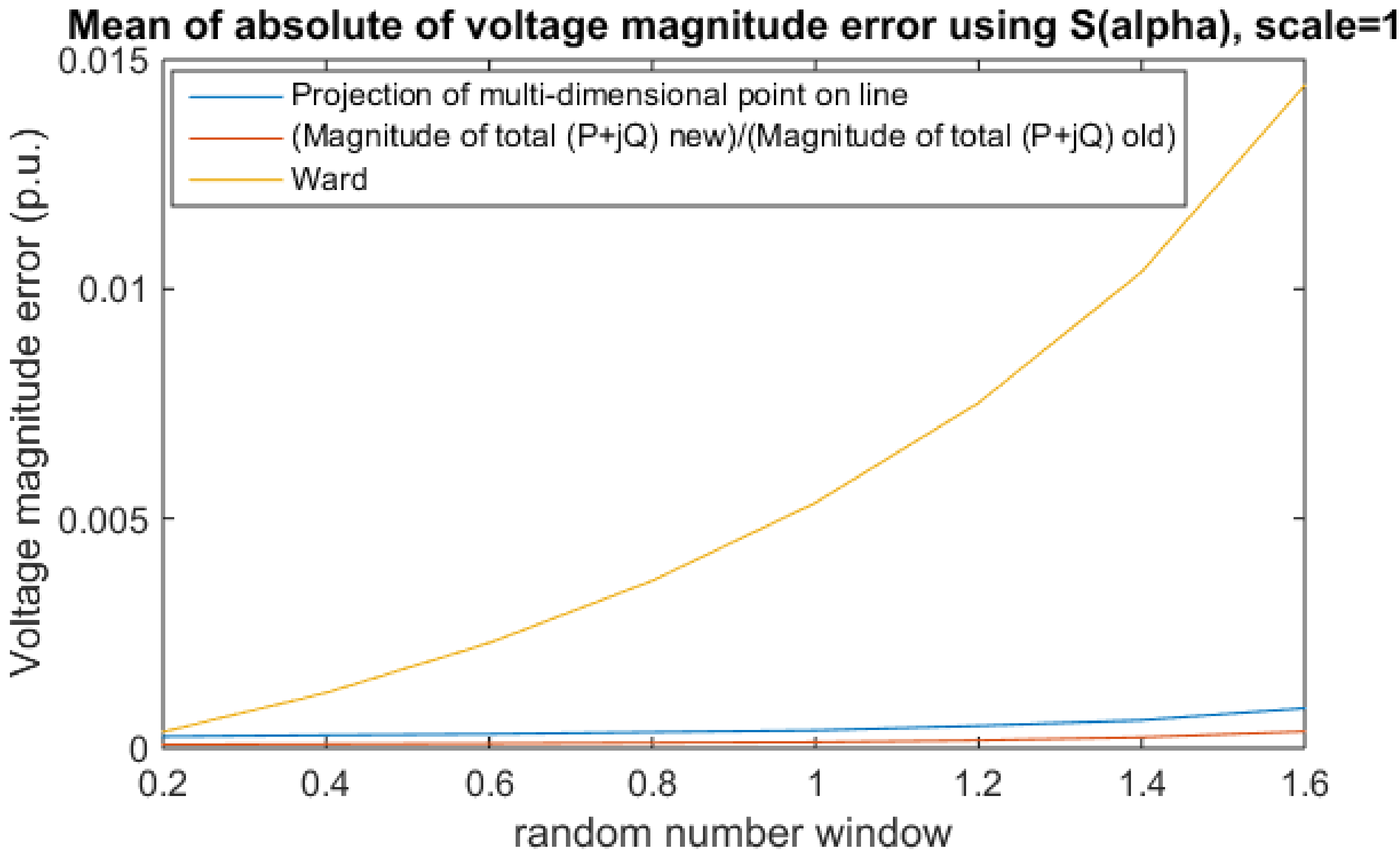


14-Bus to 2-Bus Radial Reduction Off the α Line

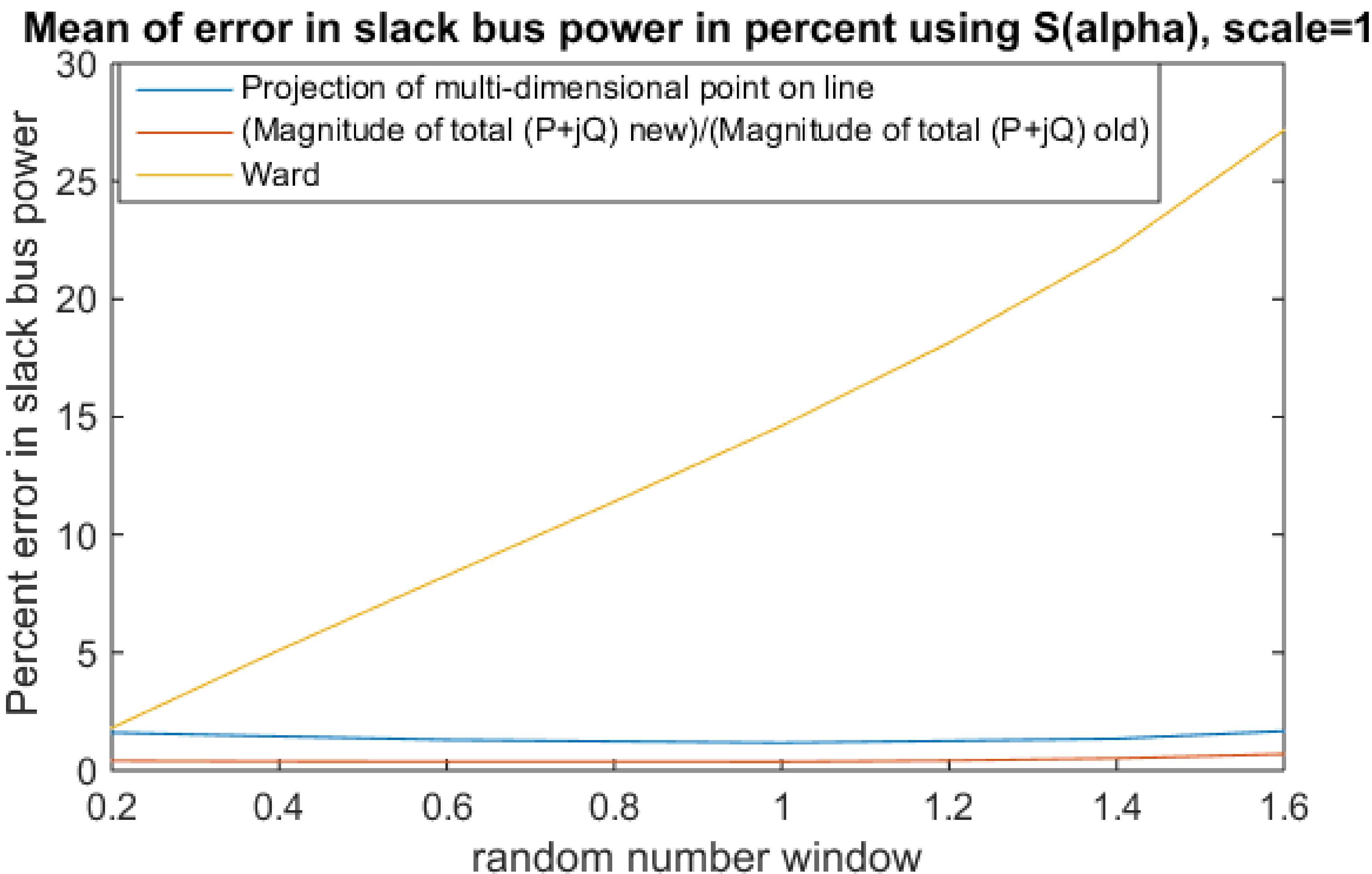
- Can HEM accurately model off the α line for larger network?
- Reduce 14-bus system to 2-bus equivalent and calculate voltage error.
- **Experiment #1**: Modify loads on the each bus at random in the following ranges: ($S_i \leftarrow S_i (1+r_i)$, r_i real, random in window range.)
  - 0-0.2
  - 0.2-0.4
  - …
  - 1.4-1.6
- HEM: Modify the load using equivalent alpha.
- Compare with Ward: Add add'l load to bus #2.

14-Bus to 2-Bus Radial Reduction Off the α Line

- Random number windows are: 0-0.2, 0.2-0.4, …, 1.4-1.6
- 1000 trials.



14-Bus to 2-Bus Radial Reduction Off the α Line



Meshed Networks
On the α Line

HEM Network Reduction
Meshed Networks

- Meshed Networks
  - Assume we have a solved base case using HEM:

$$\begin{bmatrix} Y_{ee} & Y_{eb} & \\ Y_{be} & Y_{bb} & Y_{bi} \\ & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_e(\alpha) \\ V_b(\alpha) \\ V_i(\alpha) \end{bmatrix} = \begin{bmatrix} S_e^*(\alpha) / V_e^*(\alpha^*) \\ S_b^*(\alpha) / V_b^*(\alpha^*) \\ S_i^*(\alpha) / V_i^*(\alpha^*) \end{bmatrix}$$

  - Factorize admittance matrix as if performing Ward reduction.

$$\begin{bmatrix} L_{ee} & & \\ L_{be} & I & \\ & & I \end{bmatrix} \begin{bmatrix} I & & \\ & Y_{bb} - Y_{be} Y_{ee}^{-1} Y_{eb} & Y_{bi} \\ & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} U_{ee} & U_{eb} & \\ & I & \\ & & I \end{bmatrix} \begin{bmatrix} V_e(\alpha) \\ V_b(\alpha) \\ V_i(\alpha) \end{bmatrix} = \begin{bmatrix} S_e^*(\alpha) W_e^*(\alpha^*) \\ S_b^*(\alpha) W_b^*(\alpha^*) \\ S_i^*(\alpha) W_i^*(\alpha^*) \end{bmatrix}$$

$Y_{e,b,i}$ =Admittance entry associated with external, boundary and internal system parts.
$L_{e,b,i}$ =Lower-triangular factor associated with external, boundary and internal system parts.
$U_{e,b,i}$ =Upper-triangular factor associated with external, boundary and internal system parts.
$S_{e,b,i}$ =Complex power associated with external, boundary and internal system parts.
$V_{e,b,i}$ =Bus voltage associated with external, boundary and internal system parts.
$W_{e,b,i}$ =Bus voltage inverse associated with external, boundary and internal system parts.

HEM Network Reduction
Meshed Networks

- Meshed Networks

$$\begin{bmatrix} L_{ee} & & \\ L_{be} & I & \\ & & I \end{bmatrix} \begin{bmatrix} I & & \\ & Y_{bb} - Y_{be} Y_{ee}^{-1} Y_{eb} & Y_{bi} \\ & Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} U_{ee} & U_{eb} & \\ & I & \\ & & I \end{bmatrix} \begin{bmatrix} V_e(\alpha) \\ V_b(\alpha) \\ V_i(\alpha) \end{bmatrix} = \begin{bmatrix} S_e^*(\alpha) W_e^*(\alpha^*) \\ S_b^*(\alpha) W_b^*(\alpha^*) \\ S_i^*(\alpha) W_i^*(\alpha^*) \end{bmatrix}$$

  - Multiply by inverse of L matrix and suppress external voltage values.

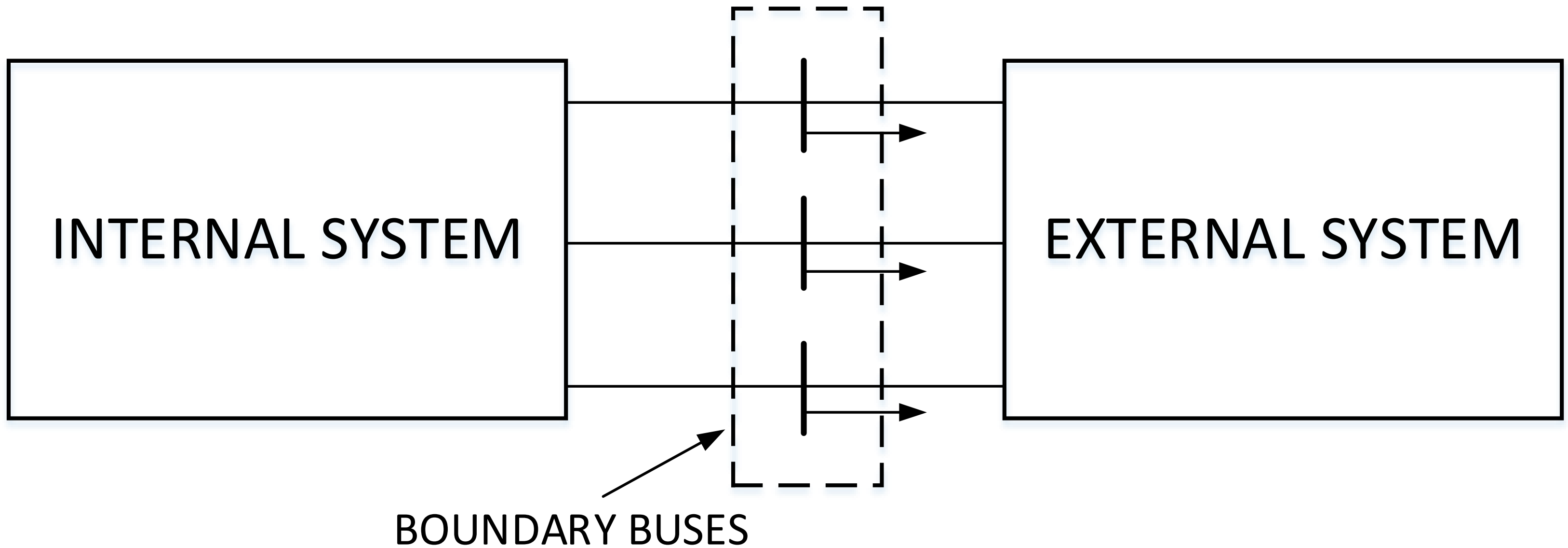
$$\begin{bmatrix} Y_{bb} - Y_{be} Y_{ee}^{-1} Y_{eb} & Y_{bi} \\ Y_{ib} & Y_{ii} \end{bmatrix} \begin{bmatrix} V_b(\alpha) \\ V_i(\alpha) \end{bmatrix} = \begin{bmatrix} \frac{S_b^*(\alpha)}{V_b^*(\alpha)} - L_{be} L_{ee}^{-1} S_e^*(\alpha) W_e^*(\alpha^*) \\ \frac{S_i^*(\alpha)}{V_i^*(\alpha^*)} \end{bmatrix}$$

Boundary bus injections

HEM Network Reduction Observations on the α line

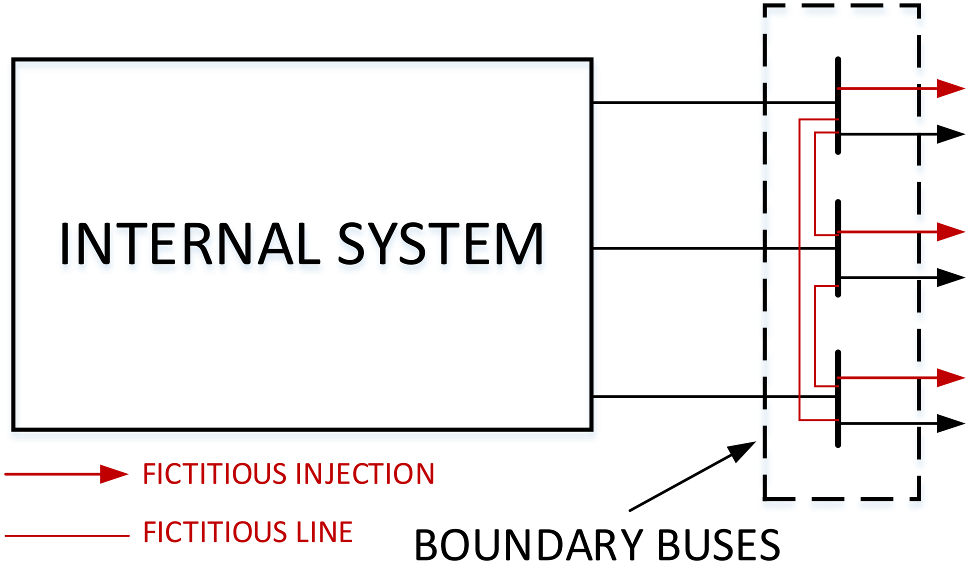
- HEM is theoretically exact on the α line provided generators don't go on var limits.
- Compare with reduced-models using methods that also model voltage/var provided by PV buses:
  - Extended Ward
  - REI

Ward Reduction



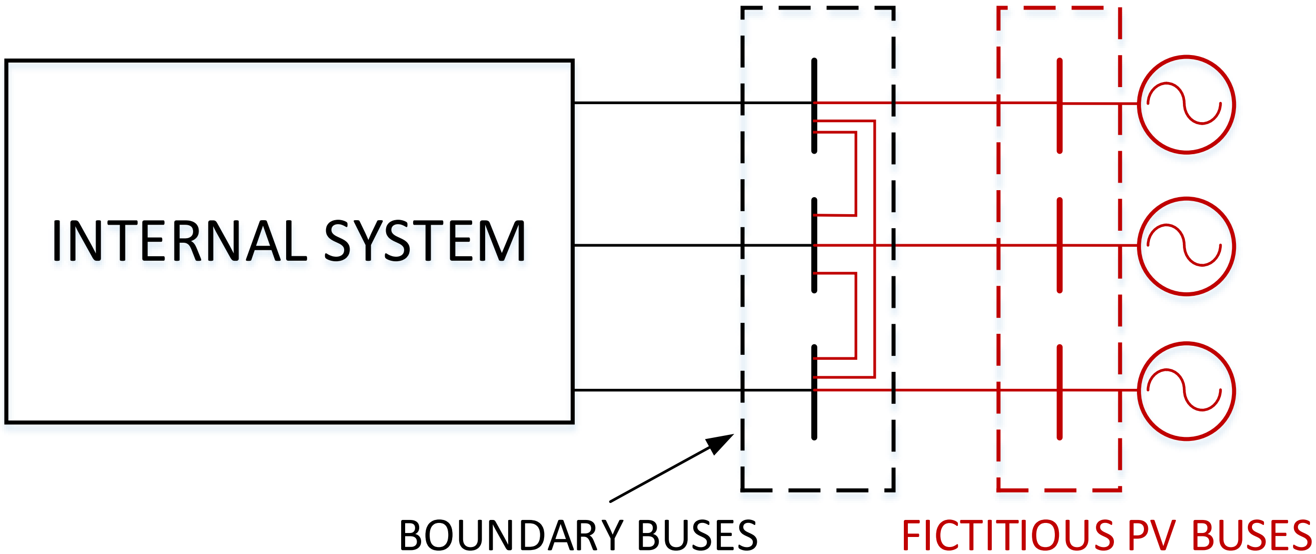
- Equivalence the external network by adding fictitious branches between boundary buses and adding fictitious injections to the boundary buses.

Ward Reduction



- Equivalence the external network by adding fictitious branches between boundary buses and adding fictitious injections to the boundary buses.
- Fictitious branches/injections created through partial LU factorization.

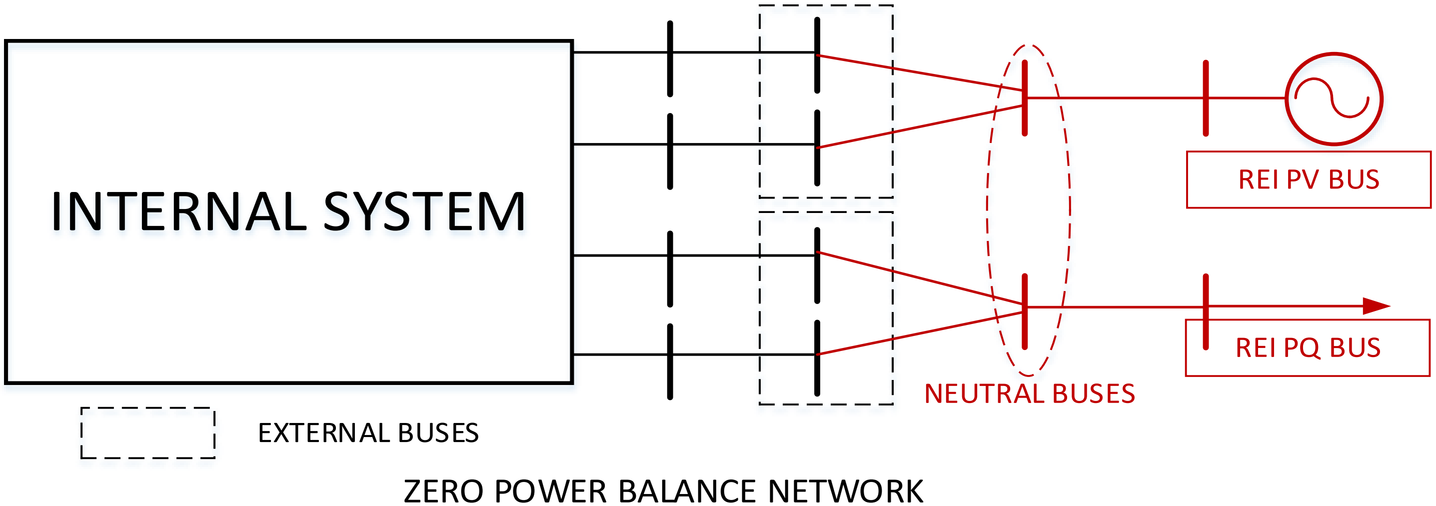
Extended Ward



- Add one fictitious generator bus to each boundary bus.
- Fictitious generator buses only provide reactive support.
- Match the incremental response for the reactive power flow.
- Generators are fictitious and have no identity, no var limits.

REI Reduction

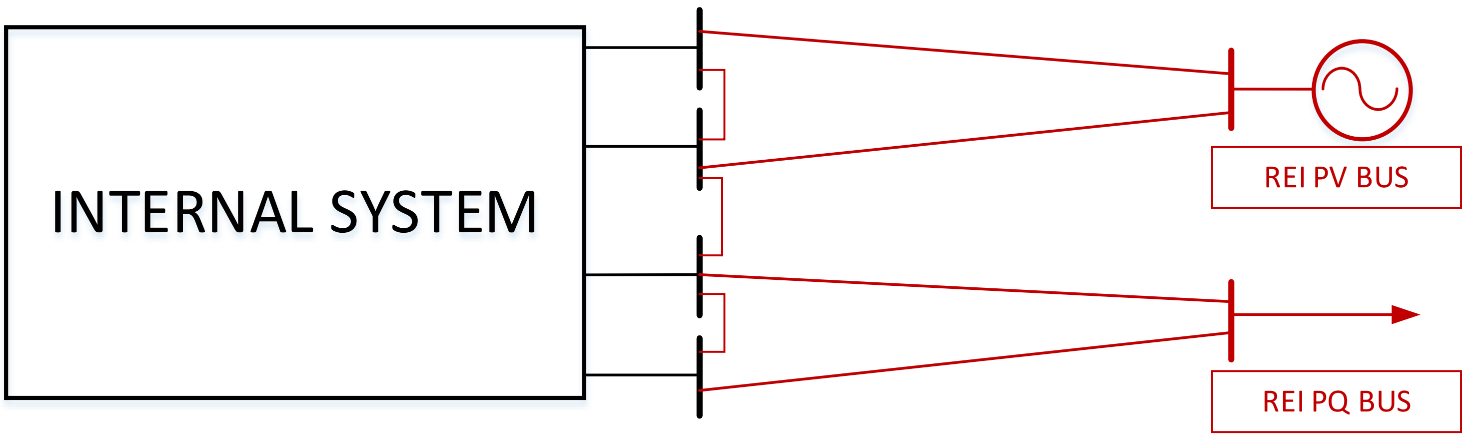
- REI (radial equivalent independent) Reduction



- Group the external buses according to type: PV/PQ.
- Create the zero power balance network.

REI Reduction

- REI (radial equivalent independent) Reduction

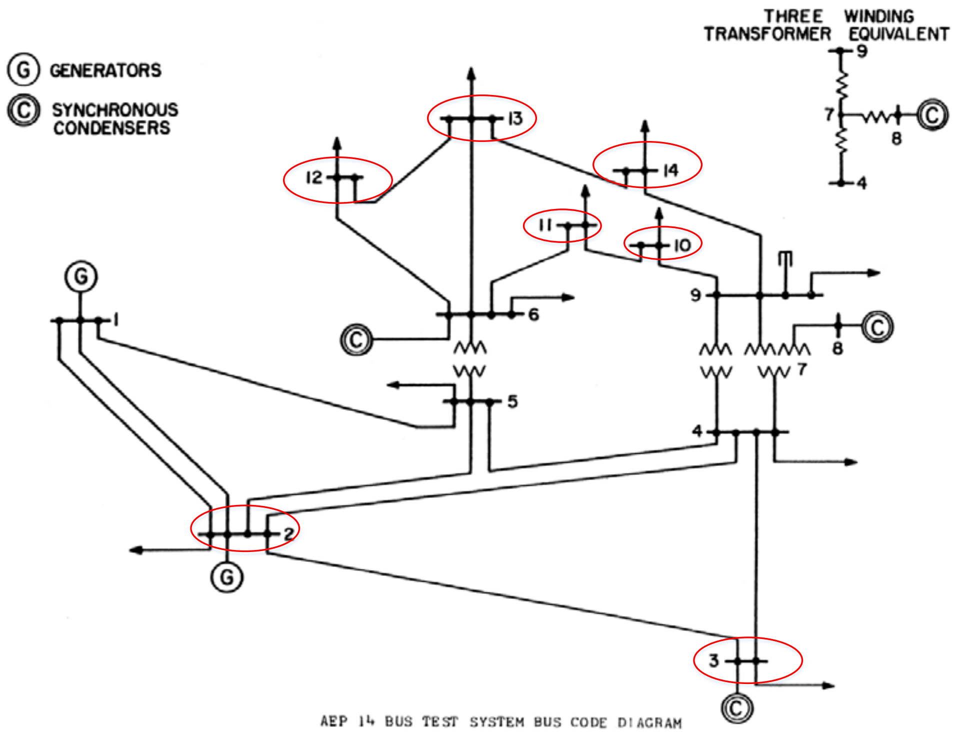


- Group the external buses according to type: PV/PQ.
- Create the zero power balance network.
- Eliminate the external buses and neutral buses to create the REI equivalent.
- Only aggregate var limits may be enforced.

HEM Network Reduction Observations on the α line

- Numerical experiments limited to the α line comparing HEM to Ward, Ex-Ward and REI. (Var limits ignored.)
  - IEEE 14 bus (7 bus internal, 7 buses external)
  - IEEE 118 bus (30 int., 88 ext.)
  - IEEE 118 bus (88 int., 30 ext.))
  - IEEE 118 bus (12 bus backbone)
  - IEEE 300 bus (89 bus backbone)

HEM Network Reduction
IEEE 14-bus schematic



- 7 external buses (red circled)—50% reduction.
- 5 boundary buses (bus 1, 4, 5, 6, 9)
- 2 internal buses (bus 7, 8)

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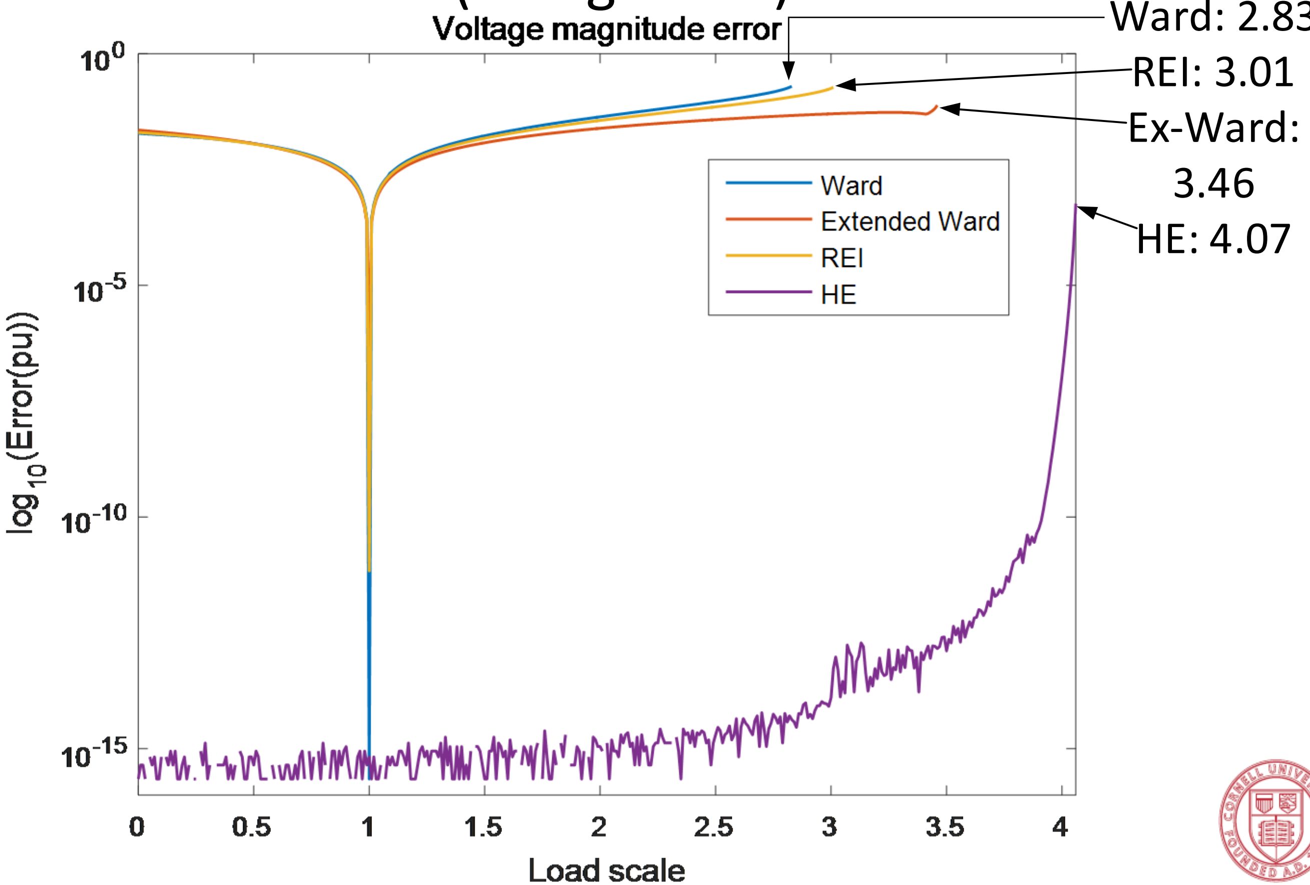
HEM Network Reduction
14 bus (7 int., 7 ext.)

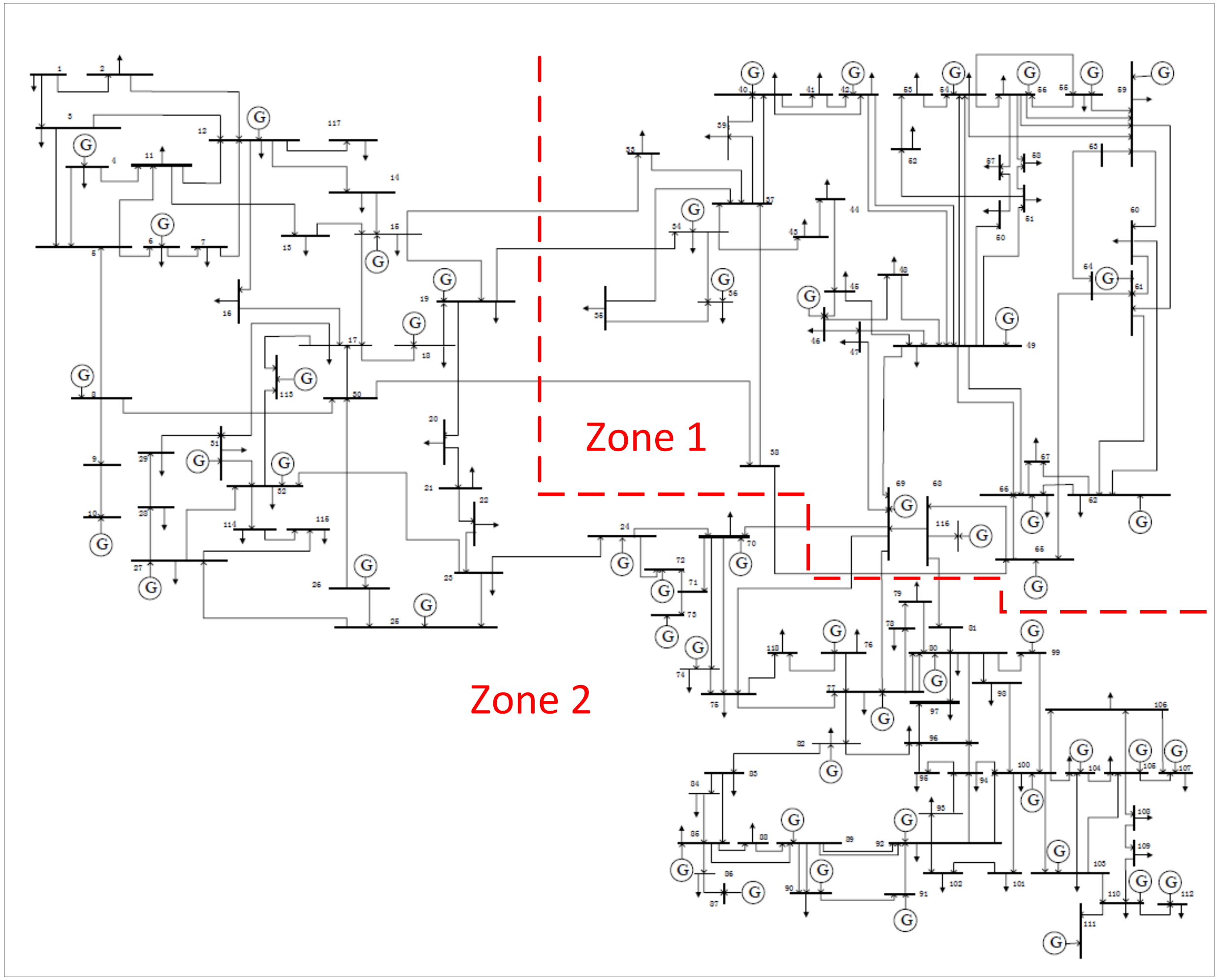
- Voltage mag. error (pu) v. α (scaled load/gen.) V_{Slack}=1.06
- Newton's method, base-case IC used to solve PF for Ward, REI, Extended Ward.
- Full model SNBP: 4.07. (Using HEM.)



HEM Network Reduction
118 bus schematic

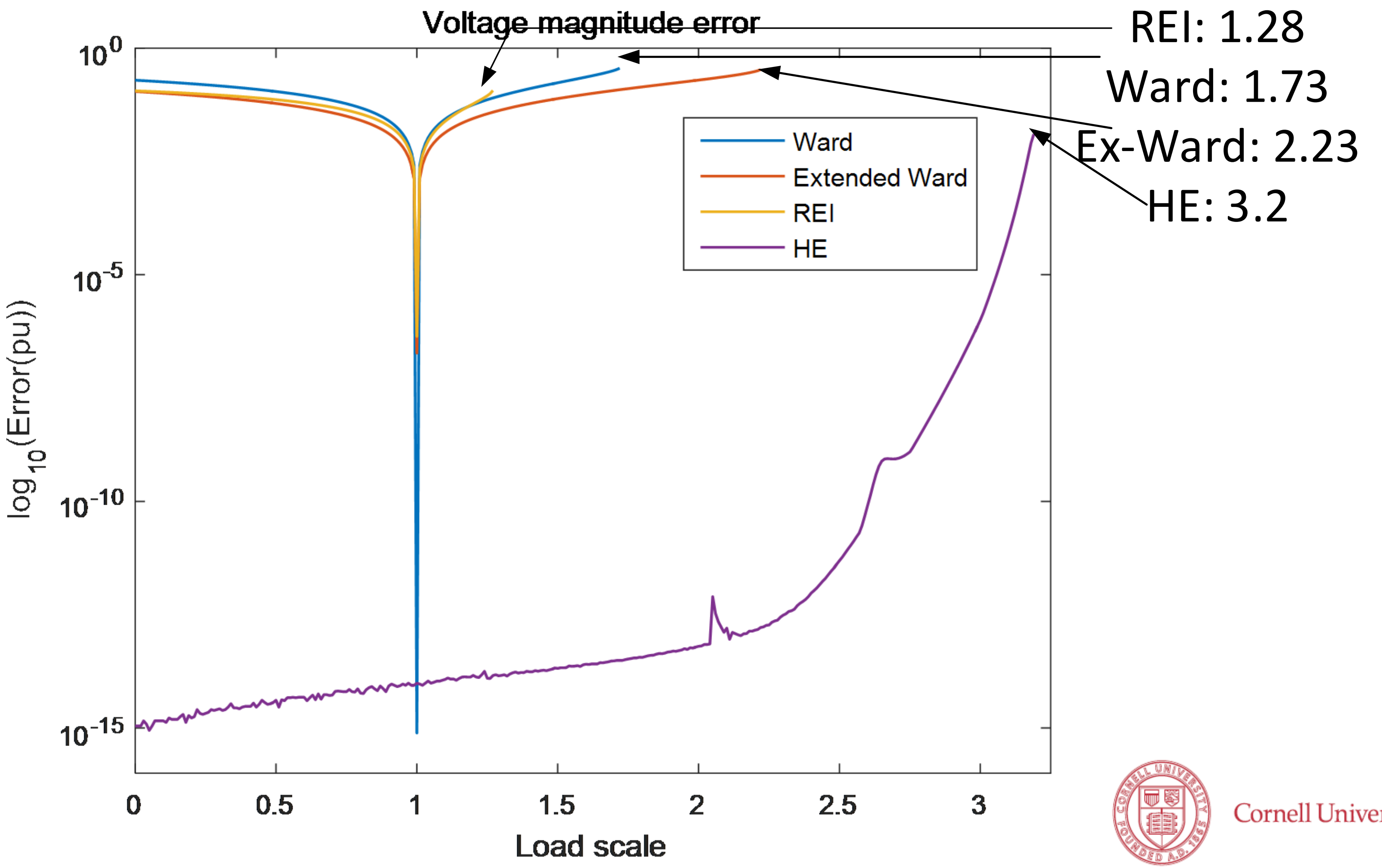
- Three reductions:

1. Preserve zone 1
2. Preserve zone 2
3. Preserve high voltage buses (>138kV)



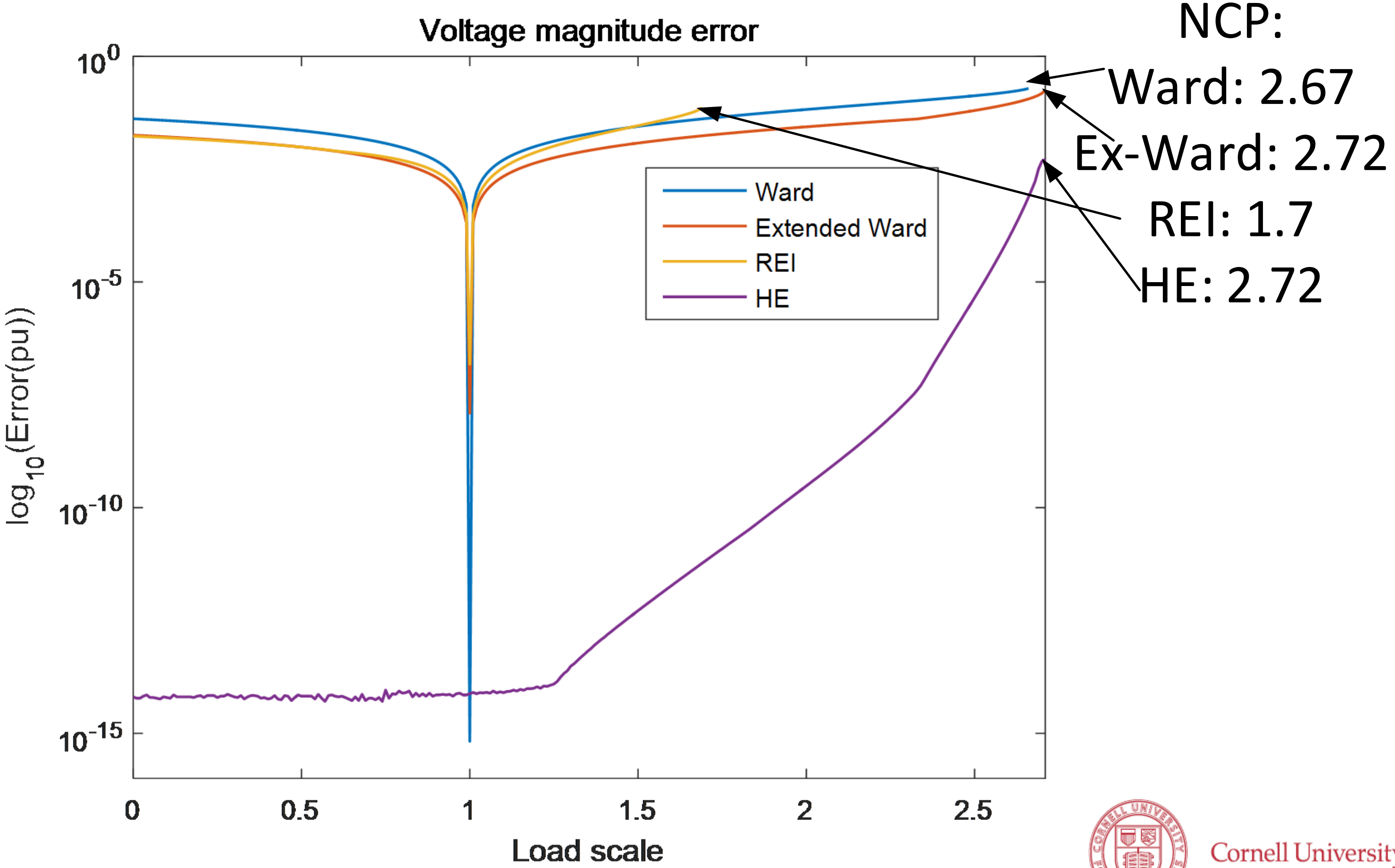
HEM Network Reduction
118 bus (30 int., 88 ext.)

- Voltage mag. error (pu) v. α (75% reduction)
- Full model SNBP=3.2.



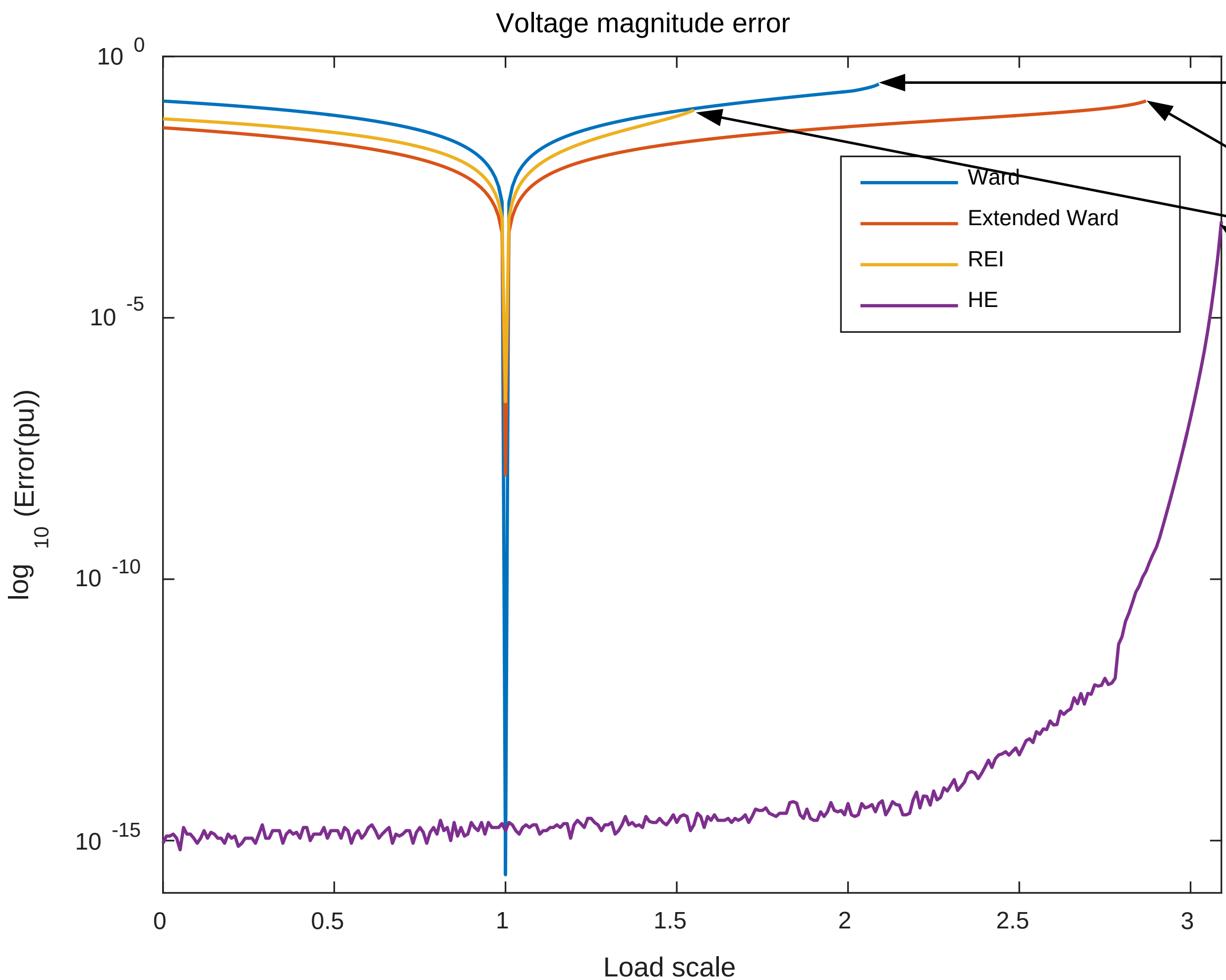
HEM Network Reduction 118 bus (88 int., 30 ext.)

- Voltage mag error (pu) v. α (25% reduction.)
- Slack bus change results in full model SNBP=2.72.



HEM Network Reduction
118 bus (12-bus backbone)

- Voltage mag error (pu) v. α (90% reduction.)
- Slack bus change: SNBP=3.1.

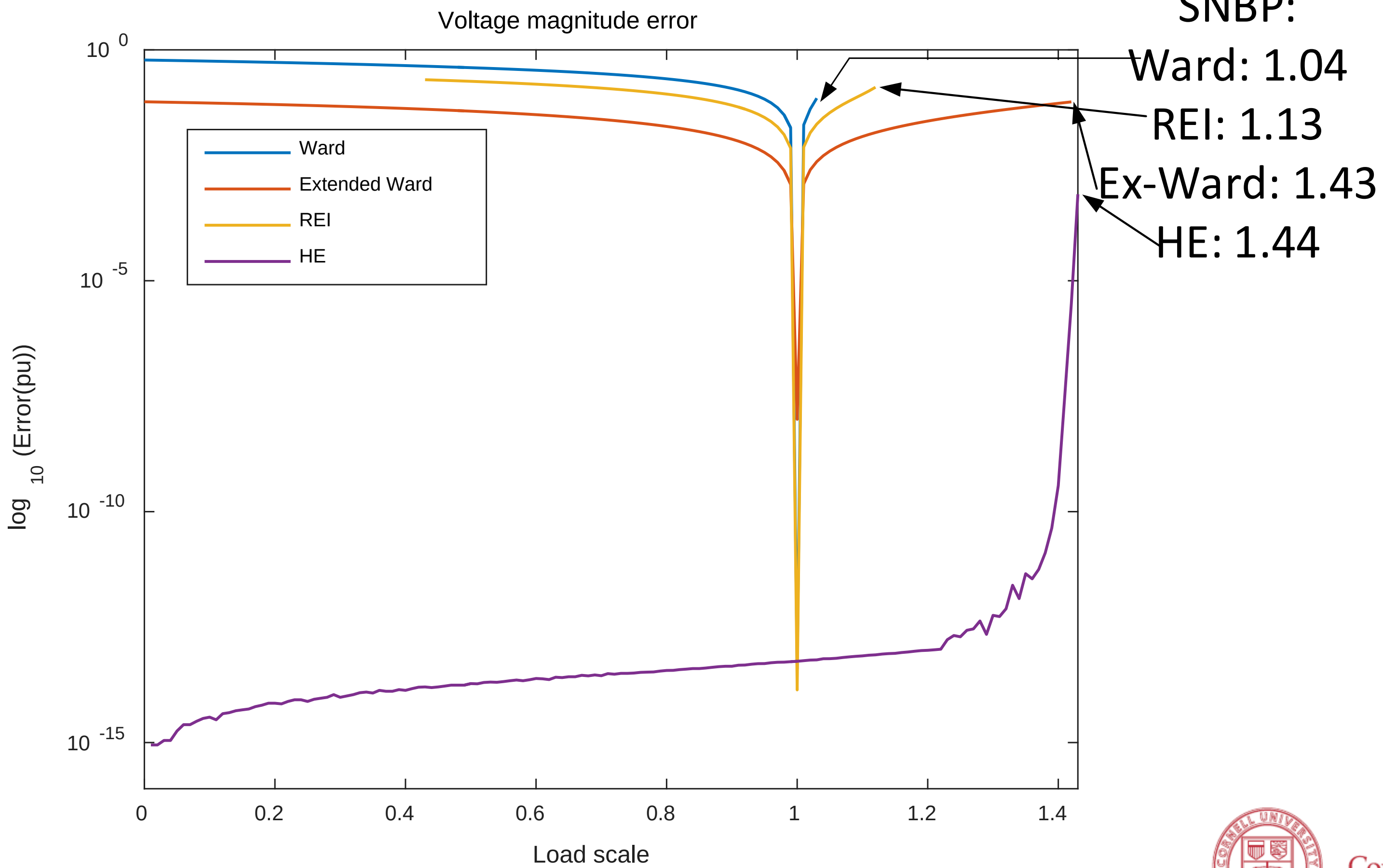


NCP:
Ward: 2.1
Ex-Ward: 2.88
REI: 1.56
HE: 3.1

HEM Network Reduction

300 bus (backbone 89 int. 211 ext.)

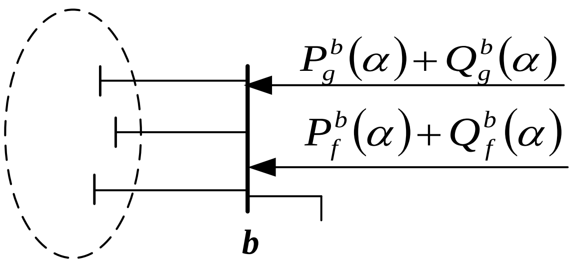
- Plot of voltage mag error (pu) v. α (70% reduction.)
- Full model SNBP: α =1.44
- Newton's method, IC=base-case solution, finds no solution for REI for 0.0<α<0.43.



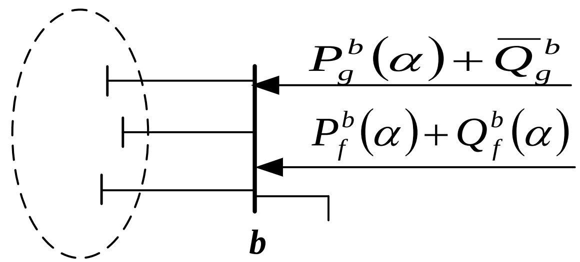
HEM Network Reduction

Imposing of var limits “on” the α line

- Initial effort.
- Perform HEM network reduction to get boundary bus injections for generators g and f: $P_g^b(\alpha) + jQ_g^b(\alpha)$; $P_f^b(\alpha) + jQ_f^b(\alpha)$.
- Let α_g be point at which generator “g” var limits, $\bar{Q}_g^b$, are encountered.



Boundary bus injections before var limits reached, $\alpha < \alpha_g$.

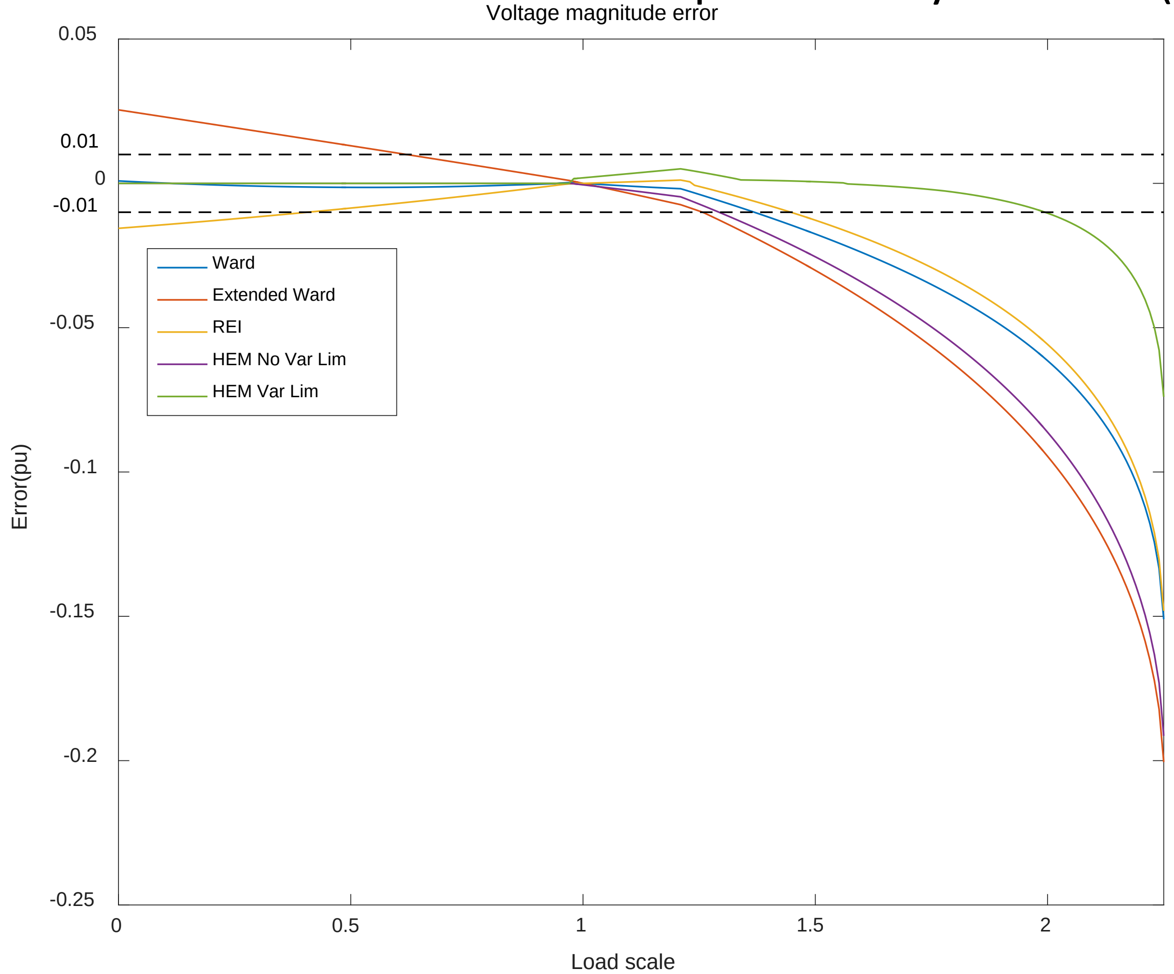


Boundary bus injections after var limits reached, $\alpha_g \leq \alpha$.

- Model with $\alpha_g < \alpha$ are approximate since all injections assume $Q_g^b(\alpha)$ behavior.
- Build reduced model with no gens on var limits: No simple way to analytically approximate effect of going off var limits.

HEM Network Reduction Imposition of var limits

- Voltage mag. error (pu) v. α (50% reduction)
- 14-bus system: Gen 2 hits var limits at α=1.34; Synch. Condenser hits var limits at α= 0.98.
- SNBP=2.25 with these var limits. 0.01 pu accuracy at α=2.0 (V_{min}=0.853 pu)



Next Steps

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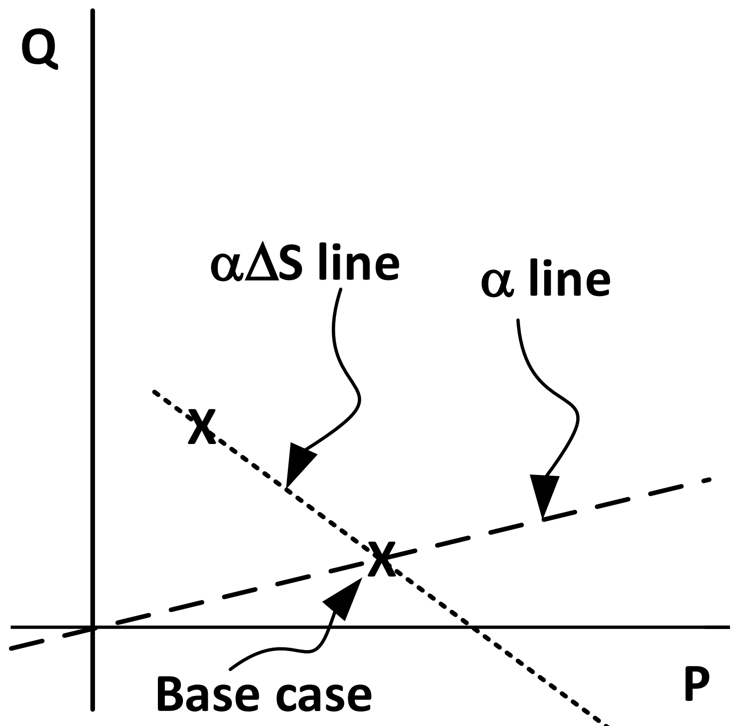
- Theory
  - Structure of nonlinear inverse injection functions.
  - Var limiting
  - Inclusion of phase shifters.
  - α line not coincident with load profile.
  - Multiple α lines, i.e., multivariate approximants, ex: Chisholm approximants.
  - Off the α line.
  - Application to the stochastic/probabilistic power flow problems.
- Numerical experimentation
  - Verify theory.

Inverse Functions for Radial Networks αΔS Line

- α line not collinear with the load/generation profile.



Dinner