## 12. Project and Program Statistics Calculations Overview

A numerical evaluation of each project within each subprogram area and a comparison to the other projects within the subprogram area necessitates a statistical comparison of the projects utilizing specific criteria. For each project, a representative set of experts in the project's field was selected to evaluate the project based upon the criteria indicated in the Introduction. Each evaluation criterion's sample mean and variance were calculated utilizing the following formulas respectively:

$$\bar{x}_{j,k} = \frac{1}{n} \sum_{i=1}^{n} x_{i,j,k}$$

$$s_{\bar{x}_{j,k}}^2 = \frac{1}{(n-1)} \sum_{i=1}^n (x_{i,j,k} - \bar{x}_{j,k})^2$$

where  $x_{i,j,k}$  is an individual reviewer's score for that criterion and n is the number of reviewers for the given project who answered the question<sup>1</sup>. The index i represents an index over the reviewers assigned for the project; the index j represents an index over the projects in that specific subprogram area; the index k represents an index over the questions asked. The sample mean for each project criterion is represented in the graph by its respective bar graph value. These calculations were performed for the numeric values supplied by the reviewers for questions 1 through 4 (those questions indicated with weight values in the Introduction).

The above values  $\bar{x}_{j,k}$  and  $s_{\bar{x}_{j,k}}^2$  can be used to extend the evaluation to the entire subprogram. In order to calculate the variance of each subprogram criterion, the sample variances must be propagated to the calculated variance of each subprogram criterion score. The subprogram area mean and variance for each evaluation criterion are then calculated as follows:

$$\bar{X}_k = \frac{1}{m} \sum_{j=1}^m \bar{x}_{j,k}$$

$$Var(\bar{X}_k) = \sigma_{\bar{X}_k}^2 + \frac{1}{m^2} \sum_{j=1}^m s_{\bar{x}_{j,k}}^2 = \frac{1}{m} \sum_{j=1}^m \bar{x}_{j,k}^2 - (\bar{X}_k)^2 + \frac{1}{m^2} \sum_{j=1}^m s_{\bar{x}_{j,k}}^2$$

where *m* is the number of projects in a subprogram area. This method of calculation allows each project to weigh evenly on each evaluation criterion of the subprogram area. The criteria means and average of the project variances values for each subprogram area (e.g., Hybrid and Vehicle Systems Technologies, Advanced Combustion Engine Technologies, Technology Integration, etc.) are represented on each project graph as the Program Area Average bullets and the red error bar ranges, respectively, for each question. In some sense, the red error bars provide a range by which projects can be evaluated by their criteria with respect to an entire subprogram area's performance. The error bar calculation was changed from the 2011 Annual Merit Review report where the expectation of the sample error was the value calculated for the error bars. This change was made so that the error bar provides a more relevant comparison for the criteria measurements of the projects to the subprogram averages.

 $<sup>^{1}</sup>$  If all of the reviewers do not answer all of the questions, the value of n will be different for some questions for a project.

Each question's score is assumed to be independent of the others for a given project (that is, for example, the question of the quality of the future research should have no bearing on the current accomplishments). Each project's weighted average score can then be calculated as follows<sup>2</sup>:

$$\bar{x}_j = \frac{\sum_{k=2}^5 w_k * \bar{x}_{j,k}}{\sum_{k=2}^5 w_k}$$

where  $w_k$  is the weight that question k has on the overall score of the  $j^{th}$  project average  $\bar{x}_j$ . The value above,  $\bar{x}_j$ , is indicated in the graphics by the Weighted Average bar. As was done for each individual project, each question's score is assumed to be independent of the others for a given subprogram. Each subprogram's weighted average score and weighted variance can then be calculated as follows:

$$\bar{X} = \frac{\sum_{k=2}^{5} w_k * \bar{X}_k}{\sum_{k=2}^{5} w_k}$$

$$Var(\bar{X}) = \frac{\sum_{k=2}^{5} w_k^2 * Var(\bar{X}_k)}{\sum_{k=2}^{5} w_k^2}$$

These values represent the Program Area Average bullet and its red error bar in the Weighted Average column.

The answers to questions 5 and 6 are represented by pie charts below the combination bar/bullet graph.

<sup>&</sup>lt;sup>2</sup> There is no need to calculate a variance for this value since it is not displayed, and it has no bearing on any future calculated value in the analysis.